UNITS, PHYSICAL QUANTITIES, AND VECTORS

By

Dr. Ammar A. AlRawi

Mathematics and
Computer Applications Department
Collage of Science
2025-2026

Goals

- To learn three fundamental quantities of physics and the units to measure them
- To keep track of significant figures in calculations
- To understand vectors and scalars and how to add vectors graphically
- To determine vector components and how to use them in calculations
- To understand unit vectors and how to use them with components to describe vectors
- To learn two ways of multiplying vectors

The nature of physics

- Physics is an *experimental* science in which physicists seek patterns that relate the phenomena of nature.
- The patterns are called *physical theories*.
- A very well established or widely used theory is called a *physical law* or *principle*.

Standards and units

- Length, time, and mass are three *fundamental* quantities of physics.
- The *International System* (SI for *System International*) is the most widely used system of units.
- In SI units, length is measured in *meters*, time in *seconds*, and mass in *kilograms*.

Unit prefixes

- Table 1.1 shows some larger and smaller units for the fundamental quantities.

Table 1.1 Some Units of Length, Mass, and Time

Length	Mass	Time
1 nanometer = 1 nm = 10^{-9} m (a few times the size of the largest atom)	1 microgram = 1 μg = 10^{-6} g = 10^{-9} kg (mass of a very small dust particle)	1 nanosecond = 1 ns = 10^{-9} s (time for light to travel 0.3 m)
1 micrometer = 1 μm = 10^{-6} m (size of some bacteria and living cells)	1 milligram = 1 mg = 10^{-3} g = 10^{-6} kg (mass of a grain of salt)	1 microsecond = 1 μs = 10^{-6} s (time for space station to move 8 mm)
1 millimeter = 1 mm = 10^{-3} m (diameter of the point of a ballpoint pen)	1 gram = 1 g = 10^{-3} kg (mass of a paper clip)	1 millisecond = 1 ms = 10^{-3} s (time for sound to travel 0.35 m)
1 centimeter = 1 cm = 10^{-2} m (diameter of your little finger)		
1 kilometer = 1 km = 10^{3} m (a 10-minute walk)		

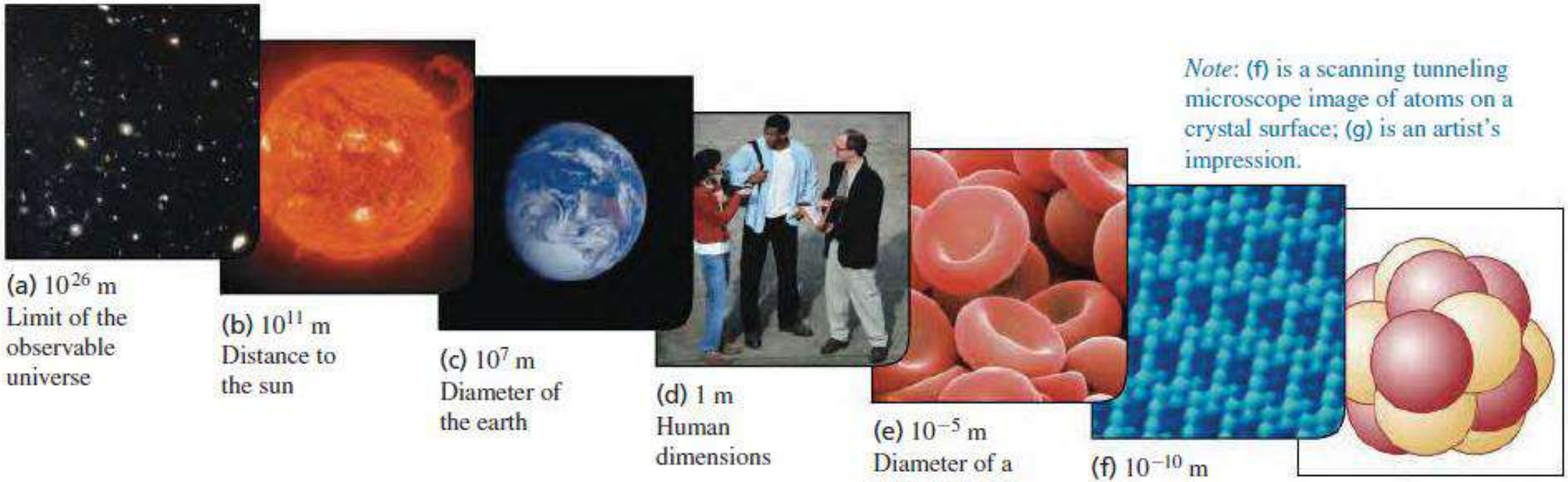
Unit prefixes

- Table 1.2 shows SI Prefixes

Table 1.2 SI Standard Prefixes

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{24}	yotta-	Y	10^{-24}	yocto-	y
10^{21}	zetta-	Z	10^{-21}	zepto-	z
10^{18}	exa-	E	10^{-18}	atto-	a
10^{15}	peta-	P	10^{-15}	femto-	f
10^{12}	tera-	T	10^{-12}	pico-	p
10^{9}	giga-	G	10^{-9}	nano-	n
10^{6}	mega-	M	10^{-6}	micro-	μ
10^{3}	kilo-	k	10^{-3}	milli-	m
10^{2}	hecto-	h	10^{-2}	centi-	c
10^{1}	deka-	da	10^{-1}	deci-	d

Some typical lengths in the universe.



(a) 10^{26} m
Limit of the observable universe

(b) 10^{11} m
Distance to the sun

(c) 10^{7} m
Diameter of the earth

(d) 1 m
Human dimensions

(e) 10^{-5} m
Diameter of a red blood cell

(f) 10^{-10} m
Radius of an atom

(g) 10^{-14} m
Radius of an atomic nucleus

Note: (f) is a scanning tunneling microscope image of atoms on a crystal surface; (g) is an artist's impression.

1.5 A speed of 7 mm/μs is equal to _____.

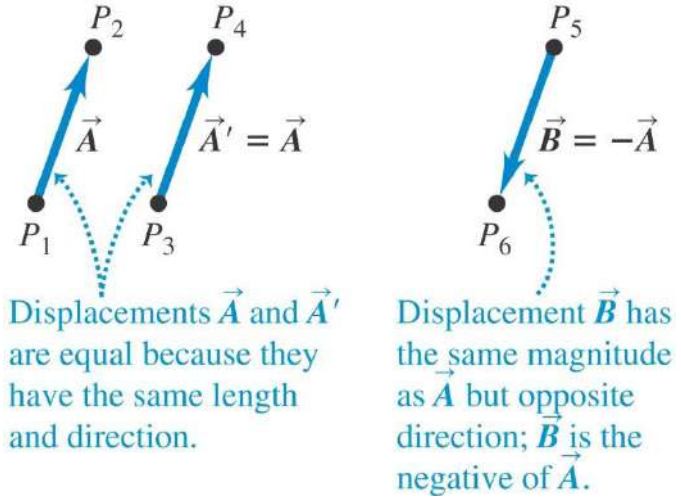
a) 7000 m/s b) 70 m/s c) 7 m/s d) 0.07 m/s

Vectors and scalars

- A *scalar quantity* can be described by a *single number*.
- A *vector quantity* has both a *magnitude* and a *direction* in space.
- In this book, a vector quantity is represented in boldface italic type with an arrow over it: $\vec{\boldsymbol{A}}$.
- The magnitude of $\vec{\boldsymbol{A}}$ is written as A or $|\vec{\boldsymbol{A}}|$.

Drawing vectors—Figure 1.10

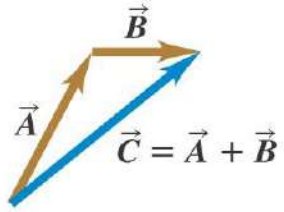
- Draw a vector as a line with an arrowhead at its tip.
- The *length* of the line shows the vector's *magnitude*.
- The *direction* of the line shows the vector's *direction*.
- Figure 1.10 shows equal-magnitude vectors having the same direction and opposite directions.



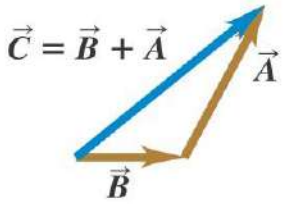
Adding two vectors graphically—Figures 1.11–1.12

- Two vectors may be added graphically using either the *parallelogram* method or the *head-to-tail* method.

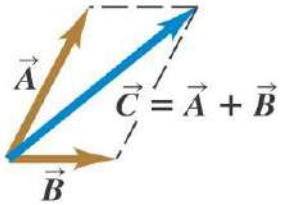
(a) We can add two vectors by placing them head to tail.



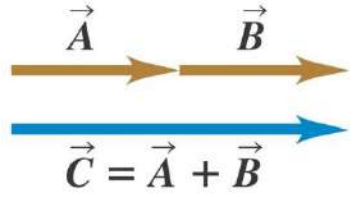
(b) Adding them in reverse order gives the same result.



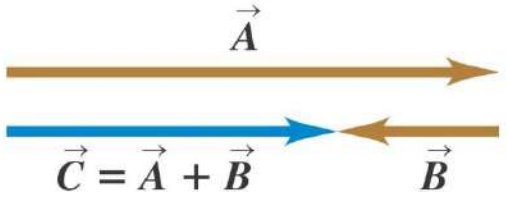
(c) We can also add them by constructing a parallelogram.



(a) The sum of two parallel vectors

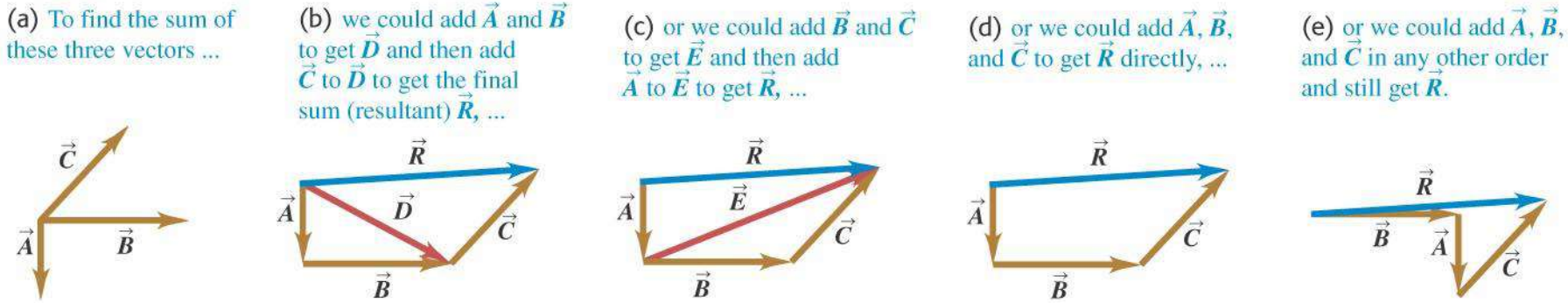


(b) The sum of two antiparallel vectors



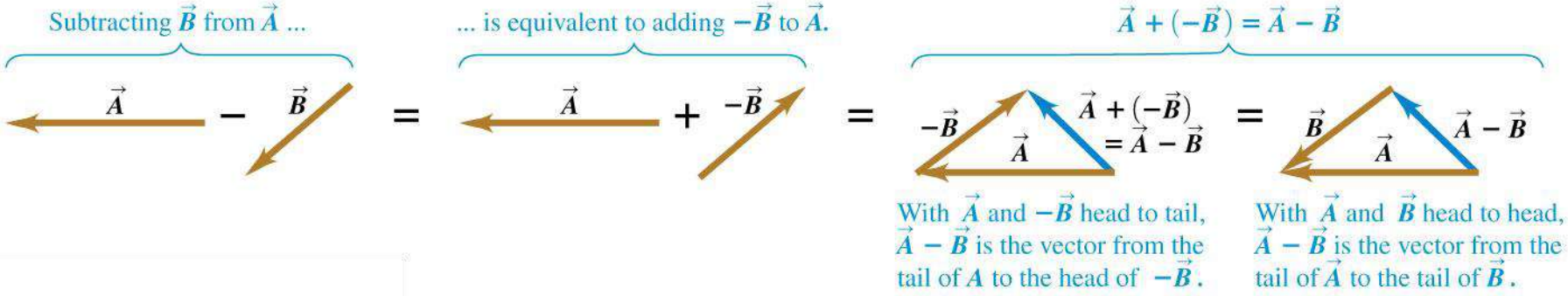
Adding more than two vectors graphically—Figure 1.13

- To add several vectors, use the head-to-tail method.
- The vectors can be added in any order.



Subtracting vectors

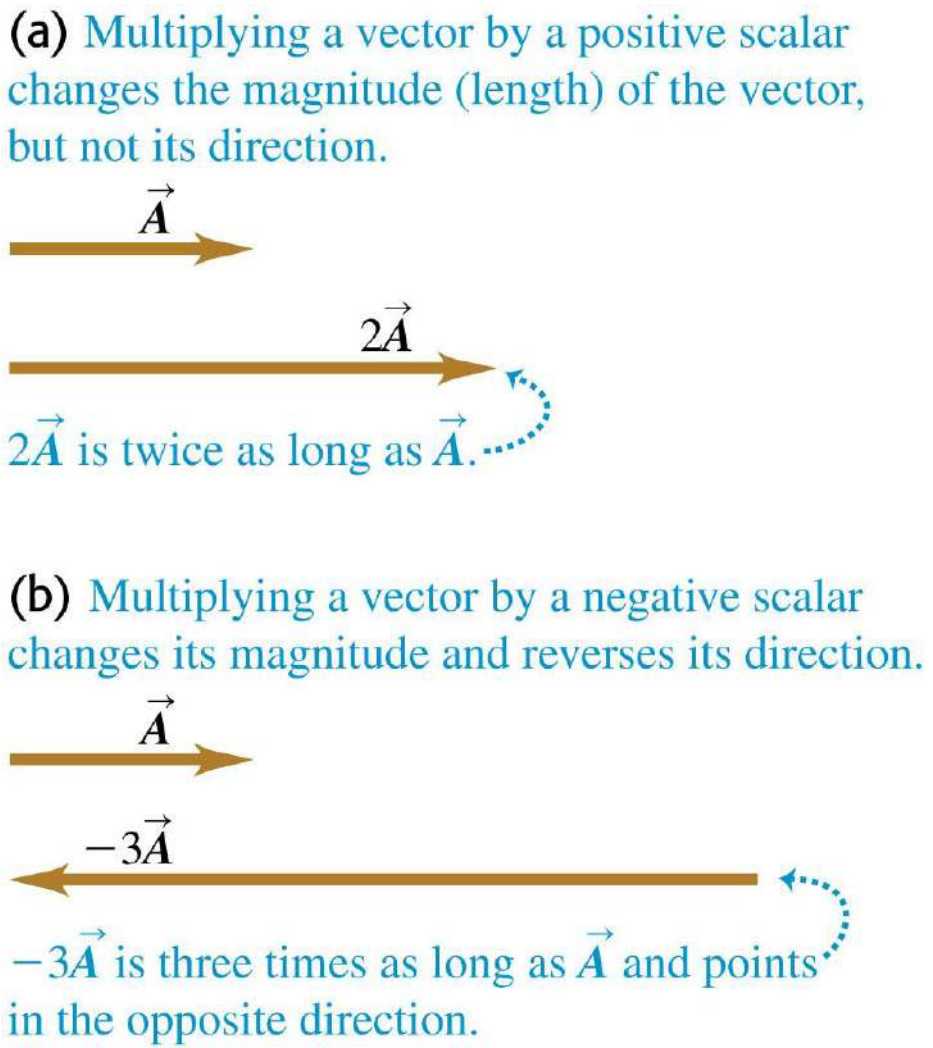
- Figure 1.14 shows how to subtract vectors.



Multiplying a vector by a scalar

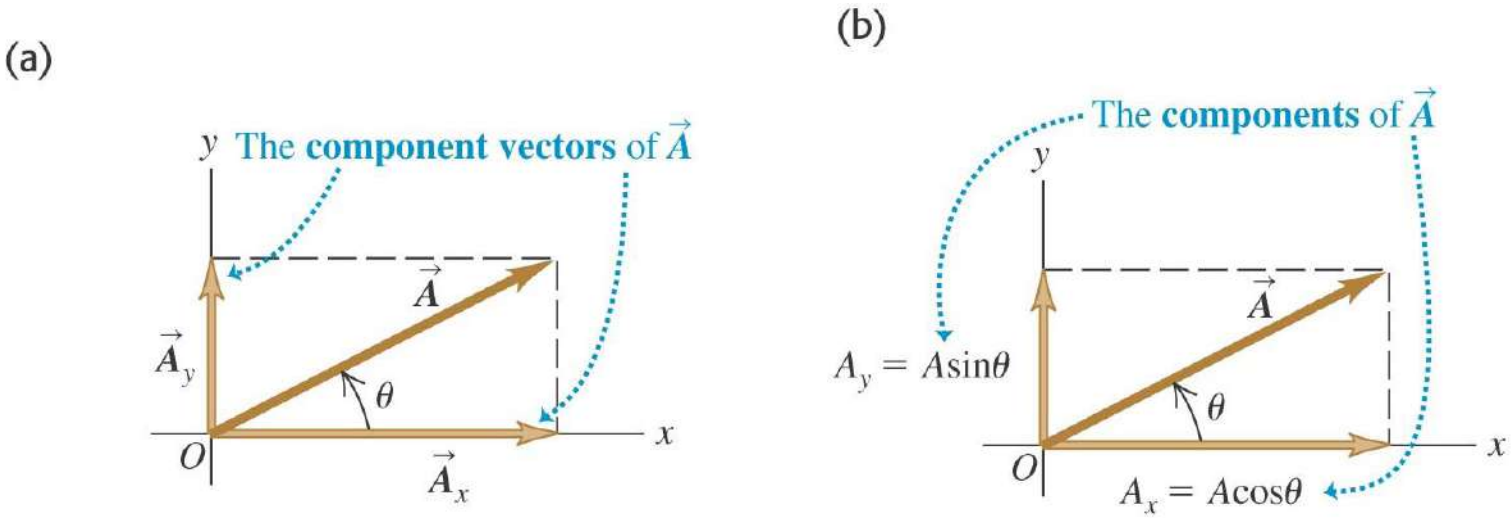
- If c is a scalar, the product $c\vec{A}$ has magnitude $|c|A$.

- Figure 1.15 illustrates multiplication of a vector by a positive scalar and a negative scalar.



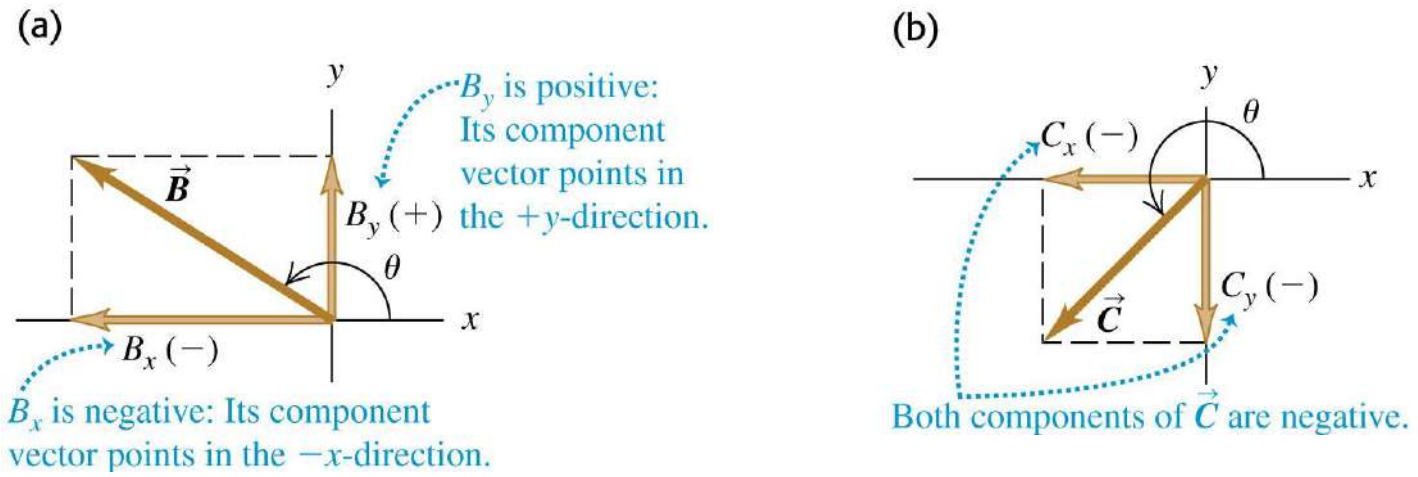
Components of a vector—Figure 1.17

- Adding vectors graphically provides limited accuracy. Vector components provide a general method for adding vectors.
- Any vector can be represented by an x-component A_x and a y-component A_y.
- Use trigonometry to find the components of a vector: $A_x = A\cos\theta$ and $A_y = A\sin\theta$, where θ is measured from the $+x$-axis toward the $+y$-axis.



Positive and negative components—Figure 1.18

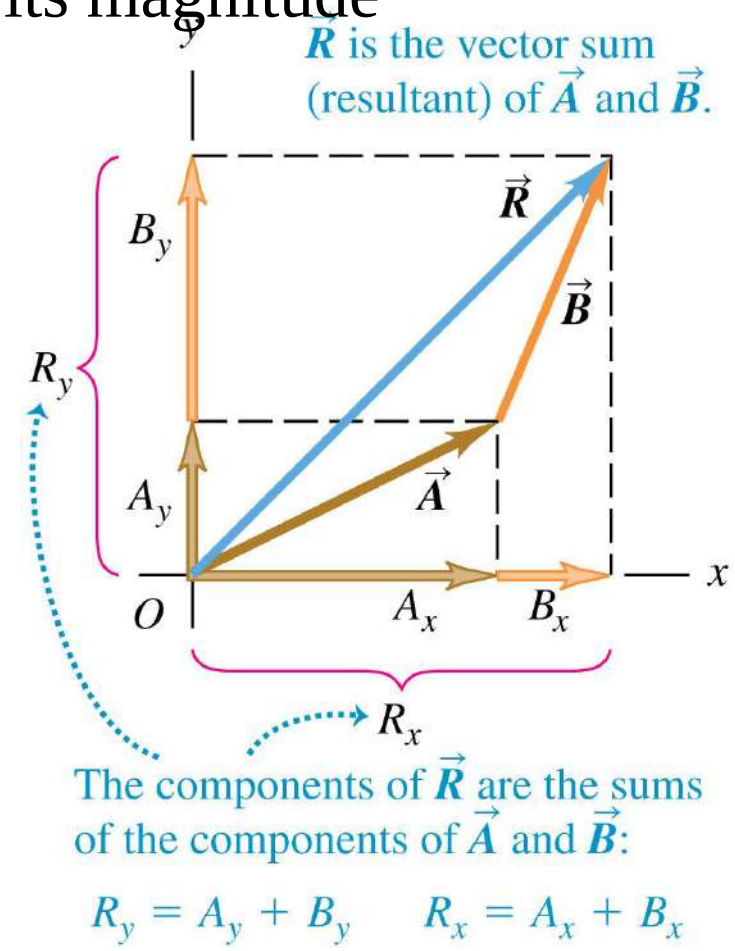
- The components of a vector can be positive or negative numbers, as shown in the figure.



Calculations using components

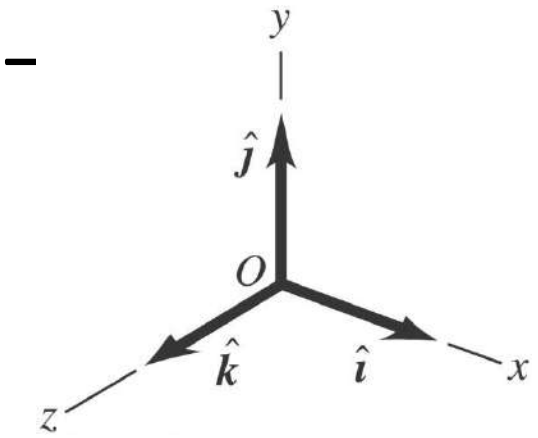
- We can use the components of a vector to find its magnitude and direction: $A=\sqrt{A_x^2+A_y^2}$ and $\tan\theta=\frac{A_y}{A_x}$
- We can use the components of a set of vectors to find the components of their sum:

$$R_x=A_x+B_x+C_x+\cdots,\quad R_y=A_y+B_y+C_y+\cdots$$

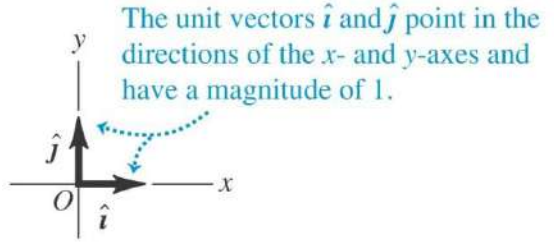


Unit vectors—Figures 1.23-

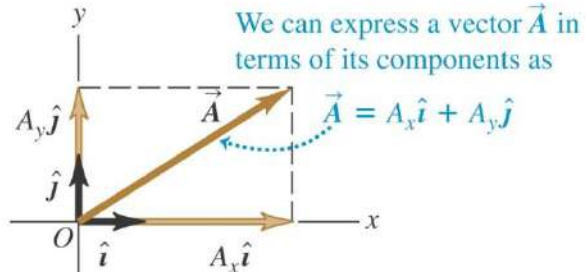
- A *unit vector* has a magnitude of 1 with no units.
- The unit vector $\hat{i}$ points in the $+x$-direction, $\hat{j}$ points in the $+y$-direction, and $\hat{k}$ points in the $+z$-direction.
- Any vector can be expressed in terms of its components as $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$.
- Follow Example 1.9.



(a)



(b)

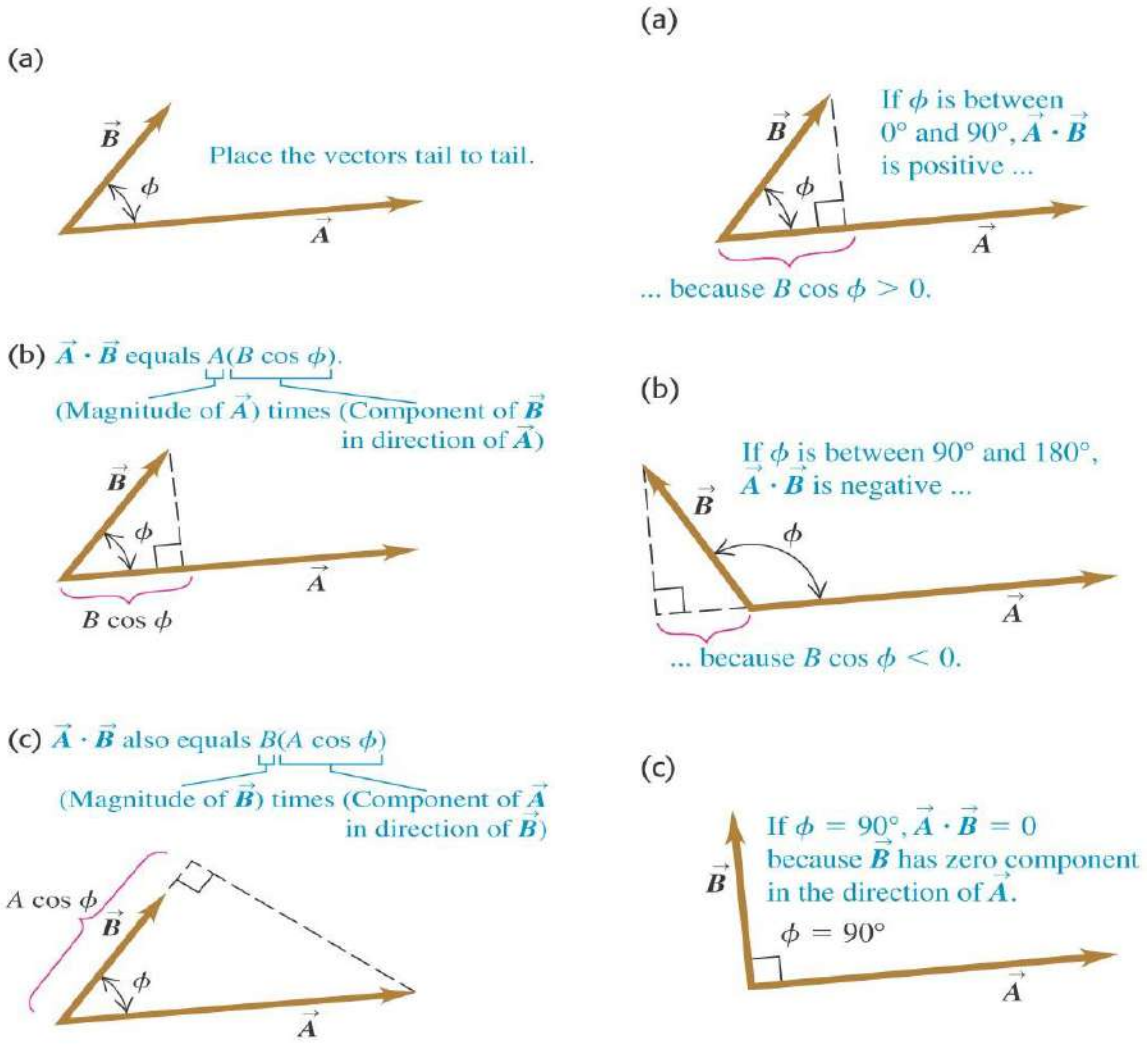


The scalar product—Figures 1.25–1.26

- The *scalar product* (also called the "dot product") of two vectors is

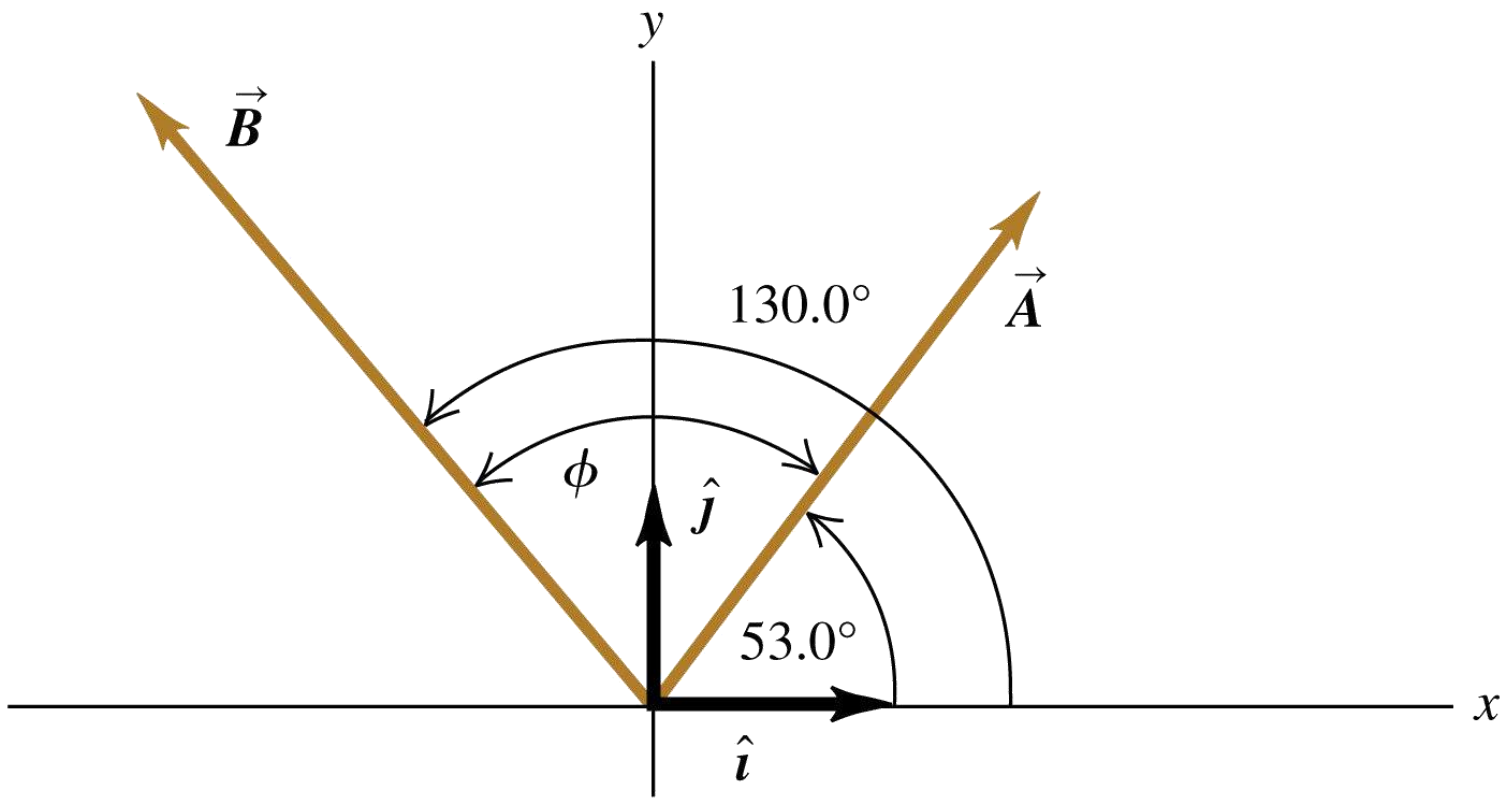
$$\vec{A}.\vec{B} = AB\cos\emptyset$$

- Figures 1.25 and 1.26 illustrate the scalar product.



Calculating a scalar product

- In terms of components, $\vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z$
- Example 1.10 shows how to calculate a scalar product in two ways.



Exercises

Q.1 A position vector has a length of 40 m and is at an angle of 57° above the x-axis. Find the vector's components?

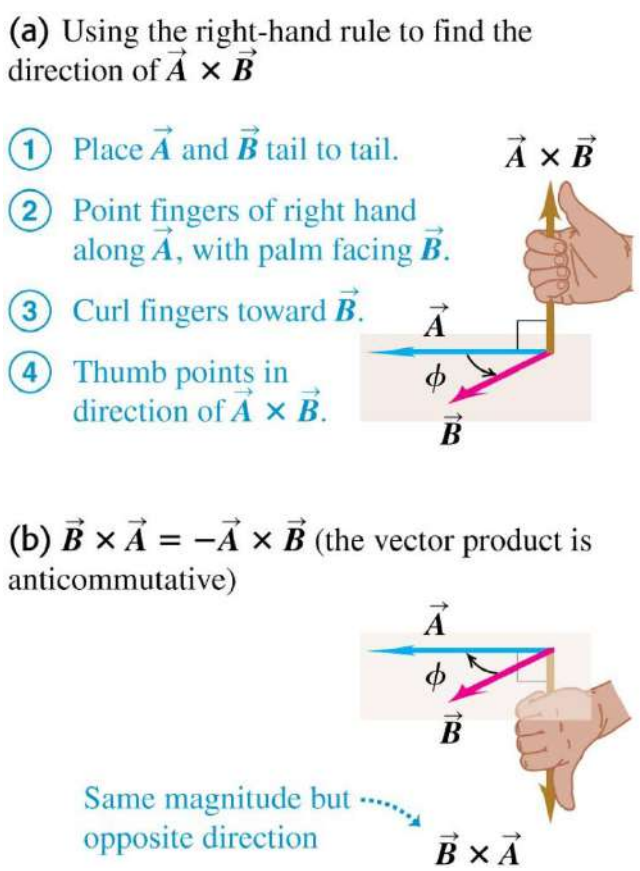
Q.2 A position vector has components x = 34.6 m and y = –53.5 m. Find the vector's length and angle with the x-axis.?

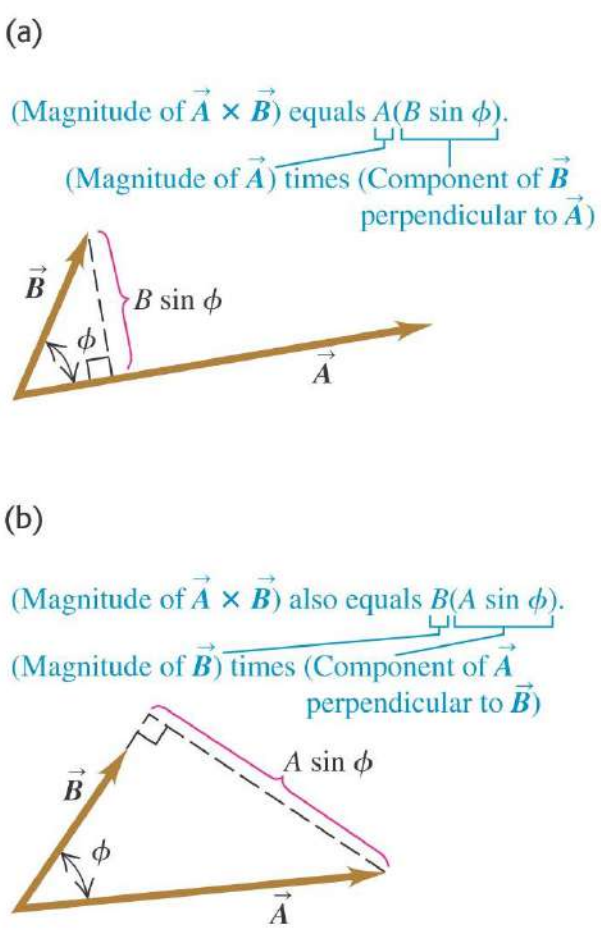
The vector product—Figures 1.29–1.30

- The vector product ("cross product") of two vectors has magnitude

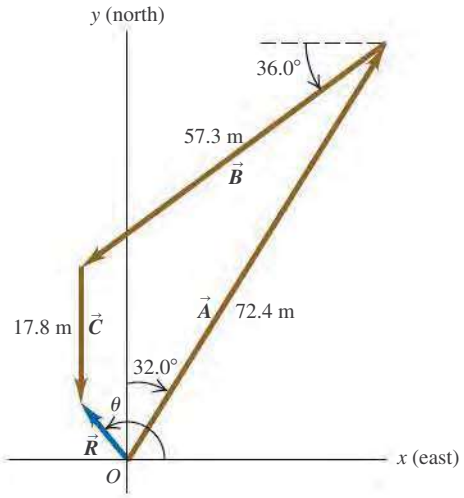
$$|\vec{A}\times\vec{B}| = AB\sin\phi$$

and the *right-hand rule* gives its direction. See Figures 1.29 and 1.30.





1.23 Three successive displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$ and the resultant (vector sum) displacement $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.



EXECUTE: The angles of the vectors, measured from the $+x$-axis toward the $+y$-axis, are $(90.0° - 32.0°) = 58.0°$, $(180.0° + 36.0°) = 216.0°$, and 270.0°, respectively. We may now use Eqs. (1.5) to find the components of $\vec{A}$:

$$A_x = A\cos\theta_A = (72.4\text{ m})(\cos 58.0°) = 38.37\text{ m}$$
$$A_y = A\sin\theta_A = (72.4\text{ m})(\sin 58.0°) = 61.40\text{ m}$$

We've kept an extra significant figure in the components; we'll round to the correct number of significant figures at the end of our calculation. The table below shows the components of all the displacements, the addition of the components, and the other calculations from Eqs. (1.6) and (1.7).

Distance	Angle	x-component	y-component
$A = 72.4$ m	58.0°	38.37 m	61.40 m
$B = 57.3$ m	216.0°	−46.36 m	−33.68 m
$C = 17.8$ m	270.0°	0.00 m	−17.80 m
		$R_x = -7.99$ m	$R_y = 9.92$ m

$$R = \sqrt{(-7.99\text{ m})^2 + (9.92\text{ m})^2} = 12.7\text{ m}$$
$$\theta = \arctan\frac{9.92\text{ m}}{-7.99\text{ m}} = -51°$$

Comparing to angle θ in Fig. 1.23 shows that the calculated angle is clearly off by 180°. The correct value is $\theta = 180° + (-51°) = 129°$, or 39° west of north.

EVALUATE: Our calculated answers for R and θ agree with our estimates. Notice how drawing the diagram in Fig. 1.23 made it easy to avoid a 180° error in the direction of the vector sum.

TEST YOUR UNDERSTANDING OF SECTION 1.8 Two vectors $\vec{A}$ and $\vec{B}$ lie in the xy-plane. (a) Can $\vec{A}$ have the same magnitude as $\vec{B}$ but different components? (b) Can $\vec{A}$ have the same components as $\vec{B}$ but a different magnitude? ❙

1.9 UNIT VECTORS

A **unit vector** is a vector that has a magnitude of 1, with no units. Its only purpose is to *point*—that is, to describe a direction in space. Unit vectors provide a convenient notation for many expressions involving components of vectors. We will always include a caret, or "hat" (^), in the symbol for a unit vector to distinguish it from ordinary vectors whose magnitude may or may not be equal to 1.

In an xy-coordinate system we can define a unit vector $\hat{\imath}$ that points in the direction of the positive x-axis and a unit vector $\hat{\jmath}$ that points in the direction of the positive y-axis (**Fig. 1.24a**). Then we can write a vector $\vec{A}$ in terms of its components as

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} \tag{1.12}$$

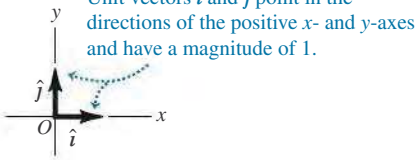
Equation (1.12) is a vector equation; each term, such as $A_x\hat{\imath}$, is a vector quantity (Fig. 1.24b).

Using unit vectors, we can express the vector sum $\vec{R}$ of two vectors $\vec{A}$ and $\vec{B}$ as follows:

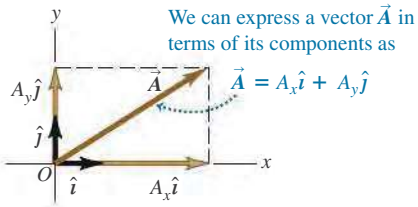
$$\begin{aligned}\vec{A} &= A_x\hat{\imath} + A_y\hat{\jmath}\\ \vec{B} &= B_x\hat{\imath} + B_y\hat{\jmath}\\ \vec{R} &= \vec{A} + \vec{B}\\ &= (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath})\\ &= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}\\ &= R_x\hat{\imath} + R_y\hat{\jmath}\end{aligned} \tag{1.13}$$

1.24 (a) The unit vectors $\hat{\imath}$ and $\hat{\jmath}$. (b) Expressing a vector $\vec{A}$ in terms of its components.

(a)



(b)



Equation (1.13) restates the content of Eqs. (1.9) in the form of a single vector equation rather than two component equations.

If not all of the vectors lie in the xy-plane, then we need a third component. We introduce a third unit vector $\hat{k}$ that points in the direction of the positive z-axis (**Fig. 1.25**). Then Eqs. (1.12) and (1.13) become

Any vector can be expressed in terms of its x-, y-, and z-components ...

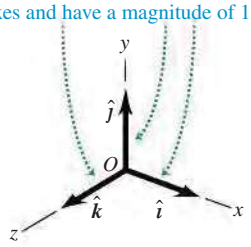
$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k} \qquad (1.14)$$

... and unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

$$\vec{R} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$
$$= R_x\hat{\imath} + R_y\hat{\jmath} + R_z\hat{k} \qquad (1.15)$$

1.25 The unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

Unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ point in the directions of the positive x-, y-, and z-axes and have a magnitude of 1.



EXAMPLE 1.8 USING UNIT VECTORS

SOLUTION

Given the two displacements

$$\vec{D} = (6.00\hat{\imath} + 3.00\hat{\jmath} - 1.00\hat{k})\text{ m} \quad \text{and}$$
$$\vec{E} = (4.00\hat{\imath} - 5.00\hat{\jmath} + 8.00\hat{k})\text{ m}$$

find the magnitude of the displacement $2\vec{D} - \vec{E}$.

SOLUTION

IDENTIFY and SET UP: We are to multiply vector $\vec{D}$ by 2 (a scalar) and subtract vector $\vec{E}$ from the result, so as to obtain the vector $\vec{F} = 2\vec{D} - \vec{E}$. Equation (1.8) says that to multiply $\vec{D}$ by 2, we multiply each of its components by 2. We can use Eq. (1.15) to do the subtraction; recall from Section 1.7 that subtracting a vector is the same as adding the negative of that vector.

EXECUTE: We have

$$\vec{F} = 2(6.00\hat{\imath} + 3.00\hat{\jmath} - 1.00\hat{k})\text{ m} - (4.00\hat{\imath} - 5.00\hat{\jmath} + 8.00\hat{k})\text{ m}$$
$$= [(12.00 - 4.00)\hat{\imath} + (6.00 + 5.00)\hat{\jmath} + (-2.00 - 8.00)\hat{k}]\text{ m}$$
$$= (8.00\hat{\imath} + 11.00\hat{\jmath} - 10.00\hat{k})\text{ m}$$

From Eq. (1.11) the magnitude of $\vec{F}$ is

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$
$$= \sqrt{(8.00\text{ m})^2 + (11.00\text{ m})^2 + (-10.00\text{ m})^2}$$
$$= 16.9\text{ m}$$

EVALUATE: Our answer is of the same order of magnitude as the larger components that appear in the sum. We wouldn't expect our answer to be much larger than this, but it could be much smaller.

TEST YOUR UNDERSTANDING OF SECTION 1.9 Arrange the following vectors in order of their magnitude, with the vector of largest magnitude first. (i) $\vec{A} = (3\hat{\imath} + 5\hat{\jmath} - 2\hat{k})$ m; (ii) $\vec{B} = (-3\hat{\imath} + 5\hat{\jmath} - 2\hat{k})$ m; (iii) $\vec{C} = (3\hat{\imath} - 5\hat{\jmath} - 2\hat{k})$ m; (iv) $\vec{D} = (3\hat{\imath} + 5\hat{\jmath} + 2\hat{k})$ m. ❙

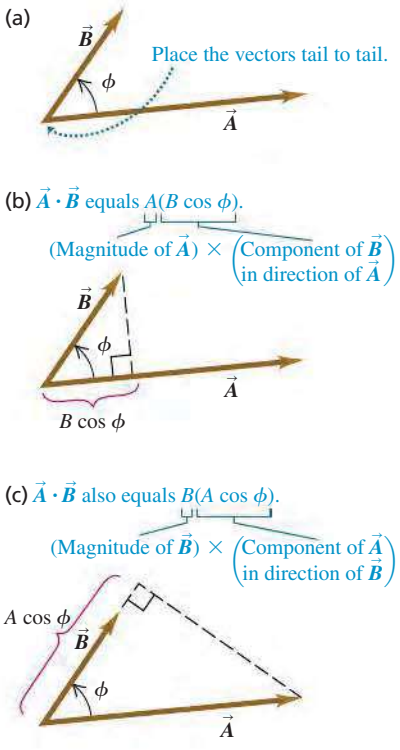
1.10 PRODUCTS OF VECTORS

We saw how vector addition develops naturally from the problem of combining displacements. It will prove useful for calculations with many other vector quantities. We can also express many physical relationships by using *products* of vectors. Vectors are not ordinary numbers, so we can't directly apply ordinary multiplication to vectors. We'll define two different kinds of products of vectors. The first, called the *scalar product,* yields a result that is a scalar quantity. The second, the *vector product,* yields another vector.

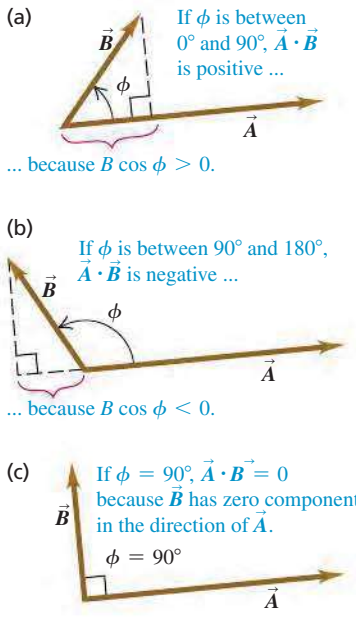
Scalar Product

We denote the **scalar product** of two vectors $\vec{A}$ and $\vec{B}$ by $\vec{A} \cdot \vec{B}$. Because of this notation, the scalar product is also called the **dot product.** Although $\vec{A}$ and $\vec{B}$ are vectors, the quantity $\vec{A} \cdot \vec{B}$ is a scalar.

1.26 Calculating the scalar product of two vectors, $\vec{A} \cdot \vec{B} = AB\cos\phi$.



1.27 The scalar product $\vec{A} \cdot \vec{B} = AB\cos\phi$ can be positive, negative, or zero, depending on the angle between $\vec{A}$ and $\vec{B}$.



To define the scalar product $\vec{A} \cdot \vec{B}$ we draw the two vectors $\vec{A}$ and $\vec{B}$ with their tails at the same point (**Fig. 1.26a**). The angle ϕ (the Greek letter phi) between their directions ranges from 0° to 180°. Figure 1.26b shows the projection of vector $\vec{B}$ onto the direction of $\vec{A}$; this projection is the component of $\vec{B}$ in the direction of $\vec{A}$ and is equal to $B\cos\phi$. (We can take components along *any* direction that's convenient, not just the x- and y-axes.) We define $\vec{A} \cdot \vec{B}$ to be the magnitude of $\vec{A}$ multiplied by the component of $\vec{B}$ in the direction of $\vec{A}$, or

Scalar (dot) product of vectors $\vec{A}$ and $\vec{B}$ — Magnitudes of $\vec{A}$ and $\vec{B}$

$$\vec{A} \cdot \vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi \qquad (1.16)$$

Angle between $\vec{A}$ and $\vec{B}$ when placed tail to tail

Alternatively, we can define $\vec{A} \cdot \vec{B}$ to be the magnitude of $\vec{B}$ multiplied by the component of $\vec{A}$ in the direction of $\vec{B}$, as in Fig. 1.26c. Hence $\vec{A} \cdot \vec{B} = B(A\cos\phi) = AB\cos\phi$, which is the same as Eq. (1.16).

The scalar product is a scalar quantity, not a vector, and it may be positive, negative, or zero. When ϕ is between 0° and 90°, $\cos\phi > 0$ and the scalar product is positive (**Fig. 1.27a**). When ϕ is between 90° and 180° so $\cos\phi < 0$, the component of $\vec{B}$ in the direction of $\vec{A}$ is negative, and $\vec{A} \cdot \vec{B}$ is negative (Fig. 1.27b). Finally, when $\phi = 90°$, $\vec{A} \cdot \vec{B} = 0$ (Fig. 1.27c). *The scalar product of two perpendicular vectors is always zero.*

For any two vectors $\vec{A}$ and $\vec{B}$, $AB\cos\phi = BA\cos\phi$. This means that $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$. The scalar product obeys the commutative law of multiplication; the order of the two vectors does not matter.

We'll use the scalar product in Chapter 6 to describe work done by a force. In later chapters we'll use the scalar product for a variety of purposes, from calculating electric potential to determining the effects that varying magnetic fields have on electric circuits.

Using Components to Calculate the Scalar Product

We can calculate the scalar product $\vec{A} \cdot \vec{B}$ directly if we know the x-, y-, and z-components of $\vec{A}$ and $\vec{B}$. To see how this is done, let's first work out the scalar products of the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$. All unit vectors have magnitude 1 and are perpendicular to each other. Using Eq. (1.16), we find

$$\begin{aligned} \hat{\imath} \cdot \hat{\imath} &= \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = (1)(1)\cos 0° = 1 \\ \hat{\imath} \cdot \hat{\jmath} &= \hat{\imath} \cdot \hat{k} = \hat{\jmath} \cdot \hat{k} = (1)(1)\cos 90° = 0 \end{aligned} \qquad (1.17)$$

Now we express $\vec{A}$ and $\vec{B}$ in terms of their components, expand the product, and use these products of unit vectors:

$$\begin{aligned} \vec{A} \cdot \vec{B} &= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \cdot (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) \\ &= A_x\hat{\imath} \cdot B_x\hat{\imath} + A_x\hat{\imath} \cdot B_y\hat{\jmath} + A_x\hat{\imath} \cdot B_z\hat{k} \\ &\quad + A_y\hat{\jmath} \cdot B_x\hat{\imath} + A_y\hat{\jmath} \cdot B_y\hat{\jmath} + A_y\hat{\jmath} \cdot B_z\hat{k} \\ &\quad + A_z\hat{k} \cdot B_x\hat{\imath} + A_z\hat{k} \cdot B_y\hat{\jmath} + A_z\hat{k} \cdot B_z\hat{k} \\ &= A_xB_x\hat{\imath} \cdot \hat{\imath} + A_xB_y\hat{\imath} \cdot \hat{\jmath} + A_xB_z\hat{\imath} \cdot \hat{k} \\ &\quad + A_yB_x\hat{\jmath} \cdot \hat{\imath} + A_yB_y\hat{\jmath} \cdot \hat{\jmath} + A_yB_z\hat{\jmath} \cdot \hat{k} \\ &\quad + A_zB_x\hat{k} \cdot \hat{\imath} + A_zB_y\hat{k} \cdot \hat{\jmath} + A_zB_z\hat{k} \cdot \hat{k} \end{aligned} \qquad (1.18)$$

From Eqs. (1.17) you can see that six of these nine terms are zero. The three that survive give

Scalar (dot) product of vectors $\vec{A}$ and $\vec{B}$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \qquad (1.19)$$

($A_x B_x$, $A_y B_y$, $A_z B_z$: Components of $\vec{A}$ and Components of $\vec{B}$)

Thus *the scalar product of two vectors is the sum of the products of their respective components.*

The scalar product gives a straightforward way to find the angle ϕ between any two vectors $\vec{A}$ and $\vec{B}$ whose components are known. In this case we can use Eq. (1.19) to find the scalar product of $\vec{A}$ and $\vec{B}$. Example 1.10 shows how to do this.

EXAMPLE 1.9 CALCULATING A SCALAR PRODUCT

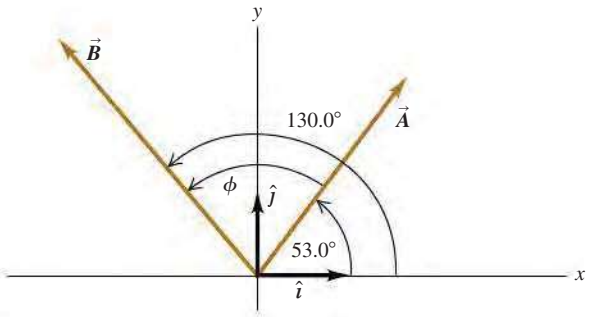
SOLUTION

Find the scalar product $\vec{A} \cdot \vec{B}$ of the two vectors in **Fig. 1.28.** The magnitudes of the vectors are $A = 4.00$ and $B = 5.00$.

SOLUTION

IDENTIFY and SET UP: We can calculate the scalar product in two ways: using the magnitudes of the vectors and the angle between them (Eq. 1.16), and using the components of the vectors (Eq. 1.19). We'll do it both ways, and the results will check each other.

1.28 Two vectors $\vec{A}$ and $\vec{B}$ in two dimensions.



EXECUTE: The angle between the two vectors $\vec{A}$ and $\vec{B}$ is $\phi = 130.0° - 53.0° = 77.0°$, so Eq. (1.16) gives us

$$\vec{A} \cdot \vec{B} = AB \cos\phi = (4.00)(5.00) \cos 77.0° = 4.50$$

To use Eq. (1.19), we must first find the components of the vectors. The angles of $\vec{A}$ and $\vec{B}$ are given with respect to the $+x$-axis and are measured in the sense from the $+x$-axis to the $+y$-axis, so we can use Eqs. (1.5):

$$A_x = (4.00) \cos 53.0° = 2.407$$
$$A_y = (4.00) \sin 53.0° = 3.195$$
$$B_x = (5.00) \cos 130.0° = -3.214$$
$$B_y = (5.00) \sin 130.0° = 3.830$$

As in Example 1.7, we keep an extra significant figure in the components and round at the end. Equation (1.19) now gives us

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$
$$= (2.407)(-3.214) + (3.195)(3.830) + (0)(0) = 4.50$$

EVALUATE: Both methods give the same result, as they should.

EXAMPLE 1.10 FINDING AN ANGLE WITH THE SCALAR PRODUCT

SOLUTION

Find the angle between the vectors

$$\vec{A} = 2.00\hat{\imath} + 3.00\hat{\jmath} + 1.00\hat{k}$$

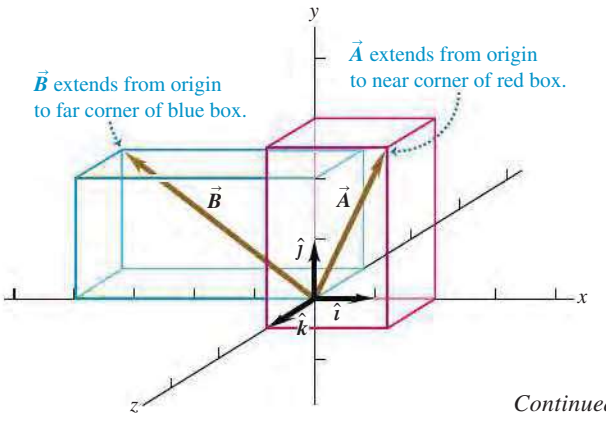
and

$$\vec{B} = -4.00\hat{\imath} + 2.00\hat{\jmath} - 1.00\hat{k}$$

SOLUTION

IDENTIFY and SET UP: We're given the x-, y-, and z-components of two vectors. Our target variable is the angle ϕ between them (**Fig. 1.29**). To find this, we'll solve Eq. (1.16), $\vec{A} \cdot \vec{B} = AB \cos\phi$, for ϕ in terms of the scalar product $\vec{A} \cdot \vec{B}$ and the magnitudes A and B. We can use Eq. (1.19) to evaluate the scalar product, $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$, and we can use Eq. (1.6) to find A and B.

1.29 Two vectors in three dimensions.



Continued

EXECUTE: We solve Eq. (1.16) for $\cos\phi$ and use Eq. (1.19) to write $\vec{A}\cdot\vec{B}$:

$$\cos\phi = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

We can use this formula to find the angle between *any* two vectors $\vec{A}$ and $\vec{B}$. Here we have $A_x = 2.00$, $A_y = 3.00$, and $A_z = 1.00$, and $B_x = -4.00$, $B_y = 2.00$, and $B_z = -1.00$. Thus

$$\begin{aligned}\vec{A}\cdot\vec{B} &= A_xB_x + A_yB_y + A_zB_z\\ &= (2.00)(-4.00) + (3.00)(2.00) + (1.00)(-1.00)\\ &= -3.00\end{aligned}$$

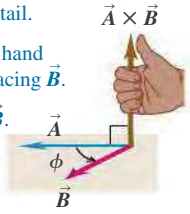
$$\begin{aligned}A &= \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(2.00)^2 + (3.00)^2 + (1.00)^2}\\ &= \sqrt{14.00}\\ B &= \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(-4.00)^2 + (2.00)^2 + (-1.00)^2}\\ &= \sqrt{21.00}\\ \cos\phi &= \frac{A_xB_x + A_yB_y + A_zB_z}{AB} = \frac{-3.00}{\sqrt{14.00}\,\sqrt{21.00}} = -0.175\\ \phi &= 100^\circ\end{aligned}$$

EVALUATE: As a check on this result, note that the scalar product $\vec{A}\cdot\vec{B}$ is negative. This means that ϕ is between 90° and 180° (see Fig. 1.27), which agrees with our answer.

1.30 The vector product of (a) $\vec{A}\times\vec{B}$ and (b) $\vec{B}\times\vec{A}$.

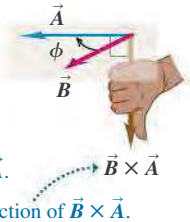
(a) Using the right-hand rule to find the direction of $\vec{A}\times\vec{B}$

1. Place $\vec{A}$ and $\vec{B}$ tail to tail.
2. Point fingers of right hand along $\vec{A}$, with palm facing $\vec{B}$.
3. Curl fingers toward $\vec{B}$.
4. Thumb points in direction of $\vec{A}\times\vec{B}$.



(b) Using the right-hand rule to find the direction of $\vec{B}\times\vec{A} = -\vec{A}\times\vec{B}$ (vector product is anticommutative)

1. Place $\vec{B}$ and $\vec{A}$ tail to tail.
2. Point fingers of right hand along $\vec{B}$, with palm facing $\vec{A}$.
3. Curl fingers toward $\vec{A}$.
4. Thumb points in direction of $\vec{B}\times\vec{A}$.
5. $\vec{B}\times\vec{A}$ has same magnitude as $\vec{A}\times\vec{B}$ but points in opposite direction.



Vector Product

We denote the **vector product** of two vectors $\vec{A}$ and $\vec{B}$, also called the **cross product,** by $\vec{A}\times\vec{B}$. As the name suggests, the vector product is itself a vector. We'll use this product in Chapter 10 to describe torque and angular momentum; in Chapters 27 and 28 we'll use it to describe magnetic fields and forces.

To define the vector product $\vec{A}\times\vec{B}$, we again draw the two vectors $\vec{A}$ and $\vec{B}$ with their tails at the same point (**Fig. 1.30a**). The two vectors then lie in a plane. We define the vector product to be a vector quantity with a direction perpendicular to this plane (that is, perpendicular to both $\vec{A}$ and $\vec{B}$) and a magnitude equal to $AB\sin\phi$. That is, if $\vec{C} = \vec{A}\times\vec{B}$, then

Magnitude of **vector (cross) product** of vectors $\vec{B}$ and $\vec{A}$

$$C = AB\sin\phi \qquad (1.20)$$

Magnitudes of $\vec{A}$ and $\vec{B}$ — Angle between $\vec{A}$ and $\vec{B}$ when placed tail to tail

We measure the angle ϕ from $\vec{A}$ toward $\vec{B}$ and take it to be the smaller of the two possible angles, so ϕ ranges from 0° to 180°. Then $\sin\phi \geq 0$ and C in Eq. (1.20) is never negative, as must be the case for a vector magnitude. Note that when $\vec{A}$ and $\vec{B}$ are parallel or antiparallel, $\phi = 0°$ or 180° and $C = 0$. That is, *the vector product of two parallel or antiparallel vectors is always zero.* In particular, *the vector product of any vector with itself is zero.*

CAUTION **Vector product vs. scalar product** Do not confuse the expression $AB\sin\phi$ for the magnitude of the vector product $\vec{A}\times\vec{B}$ with the similar expression $AB\cos\phi$ for the scalar product $\vec{A}\cdot\vec{B}$. To see the difference between these two expressions, imagine that we vary the angle between $\vec{A}$ and $\vec{B}$ while keeping their magnitudes constant. When $\vec{A}$ and $\vec{B}$ are parallel, the magnitude of the vector product will be zero and the scalar product will be maximum. When $\vec{A}$ and $\vec{B}$ are perpendicular, the magnitude of the vector product will be maximum and the scalar product will be zero.

There are always *two* directions perpendicular to a given plane, one on each side of the plane. We choose which of these is the direction of $\vec{A}\times\vec{B}$ as follows. Imagine rotating vector $\vec{A}$ about the perpendicular line until $\vec{A}$ is aligned with $\vec{B}$, choosing the smaller of the two possible angles between $\vec{A}$ and $\vec{B}$. Curl the fingers of your right hand around the perpendicular line so that your fingertips point in the direction of rotation; your thumb will then point in the direction of $\vec{A}\times\vec{B}$. Figure 1.30a shows this **right-hand rule** and describes a second way to think about this rule.

Similarly, we determine the direction of $\vec{B} \times \vec{A}$ by rotating $\vec{B}$ into $\vec{A}$ as in Fig. 1.30b. The result is a vector that is *opposite* to the vector $\vec{A} \times \vec{B}$. The vector product is *not* commutative but instead is *anticommutative:* For any two vectors $\vec{A}$ and $\vec{B}$,

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \tag{1.21}$$

Just as we did for the scalar product, we can give a geometrical interpretation of the magnitude of the vector product. In **Fig. 1.31a,** $B \sin\phi$ is the component of vector $\vec{B}$ that is *perpendicular* to the direction of vector $\vec{A}$. From Eq. (1.20) the magnitude of $\vec{A} \times \vec{B}$ equals the magnitude of $\vec{A}$ multiplied by the component of $\vec{B}$ that is perpendicular to $\vec{A}$. Figure 1.31b shows that the magnitude of $\vec{A} \times \vec{B}$ also equals the magnitude of $\vec{B}$ multiplied by the component of $\vec{A}$ that is perpendicular to $\vec{B}$. Note that Fig. 1.31 shows the case in which ϕ is between 0° and 90°; draw a similar diagram for ϕ between 90° and 180° to show that the same geometrical interpretation of the magnitude of $\vec{A} \times \vec{B}$ applies.

Using Components to Calculate the Vector Product

If we know the components of $\vec{A}$ and $\vec{B}$, we can calculate the components of the vector product by using a procedure similar to that for the scalar product. First we work out the multiplication table for unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$, all three of which are perpendicular to each other (**Fig. 1.32a**). The vector product of any vector with itself is zero, so

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \mathbf{0}$$

The boldface zero is a reminder that each product is a zero *vector*—that is, one with all components equal to zero and an undefined direction. Using Eqs. (1.20) and (1.21) and the right-hand rule, we find

$$\begin{aligned} \hat{\imath} \times \hat{\jmath} &= -\hat{\jmath} \times \hat{\imath} = \hat{k} \\ \hat{\jmath} \times \hat{k} &= -\hat{k} \times \hat{\jmath} = \hat{\imath} \\ \hat{k} \times \hat{\imath} &= -\hat{\imath} \times \hat{k} = \hat{\jmath} \end{aligned} \tag{1.22}$$

You can verify these equations by referring to Fig. 1.32a.

Next we express $\vec{A}$ and $\vec{B}$ in terms of their components and the corresponding unit vectors, and we expand the expression for the vector product:

$$\begin{aligned} \vec{A} \times \vec{B} &= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) \\ &= A_x\hat{\imath} \times B_x\hat{\imath} + A_x\hat{\imath} \times B_y\hat{\jmath} + A_x\hat{\imath} \times B_z\hat{k} \\ &\quad + A_y\hat{\jmath} \times B_x\hat{\imath} + A_y\hat{\jmath} \times B_y\hat{\jmath} + A_y\hat{\jmath} \times B_z\hat{k} \\ &\quad + A_z\hat{k} \times B_x\hat{\imath} + A_z\hat{k} \times B_y\hat{\jmath} + A_z\hat{k} \times B_z\hat{k} \end{aligned} \tag{1.23}$$

We can also rewrite the individual terms in Eq. (1.23) as $A_x\hat{\imath} \times B_y\hat{\jmath} = (A_xB_y)\hat{\imath} \times \hat{\jmath}$, and so on. Evaluating these by using the multiplication table for the unit vectors in Eqs. (1.22) and then grouping the terms, we get

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k} \tag{1.24}$$

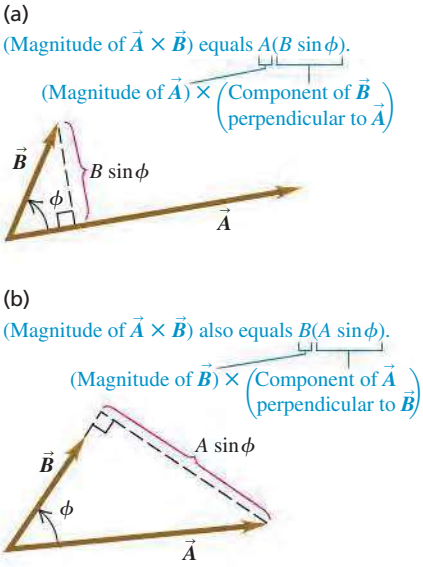
If you compare Eq. (1.24) with Eq. (1.14), you'll see that the components of $\vec{C} = \vec{A} \times \vec{B}$ are

Components of vector (cross) product $\vec{A} \times \vec{B}$

$$C_x = A_yB_z - A_zB_y \qquad C_y = A_zB_x - A_xB_z \qquad C_z = A_xB_y - A_yB_x \tag{1.25}$$

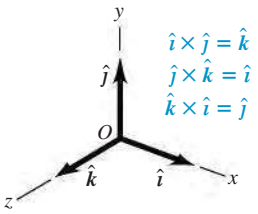
A_x, A_y, A_z = components of $\vec{A}$ $\qquad$ B_x, B_y, B_z = components of $\vec{B}$

1.31 Calculating the magnitude $AB\sin\phi$ of the vector product of two vectors, $\vec{A} \times \vec{B}$.

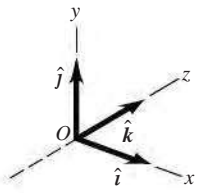


1.32 (a) We will always use a right-handed coordinate system, like this one. (b) We will never use a left-handed coordinate system (in which $\hat{\imath} \times \hat{\jmath} = -\hat{k}$, and so on).

(a) A right-handed coordinate system



(b) A left-handed coordinate system; we will not use these.



With the axis system of Fig. 1.32a, if we reverse the direction of the z-axis, we get the system shown in Fig. 1.32b. Then, as you may verify, the definition of the vector product gives $\hat{\imath} \times \hat{\jmath} = -\hat{k}$ instead of $\hat{\imath} \times \hat{\jmath} = \hat{k}$. In fact, all vector products of unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ would have signs opposite to those in Eqs. (1.22). So there are two kinds of coordinate systems, which differ in the signs of the vector products of unit vectors. An axis system in which $\hat{\imath} \times \hat{\jmath} = \hat{k}$, as in Fig. 1.32a, is called a **right-handed system.** The usual practice is to use *only* right-handed systems, and we'll follow that practice throughout this book.

EXAMPLE 1.11 CALCULATING A VECTOR PRODUCT

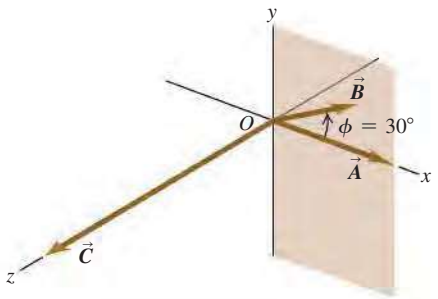
SOLUTION

Vector $\vec{A}$ has magnitude 6 units and is in the direction of the $+x$-axis. Vector $\vec{B}$ has magnitude 4 units and lies in the xy-plane, making an angle of 30° with the $+x$-axis (**Fig. 1.33**). Find the vector product $\vec{C} = \vec{A} \times \vec{B}$.

SOLUTION

IDENTIFY and SET UP: We'll find the vector product in two ways, which will provide a check of our calculations. First we'll use Eq. (1.20) and the right-hand rule; then we'll use Eqs. (1.25) to find the vector product by using components.

1.33 Vectors $\vec{A}$ and $\vec{B}$ and their vector product $\vec{C} = \vec{A} \times \vec{B}$. Vector $\vec{B}$ lies in the xy-plane.



EXECUTE: From Eq. (1.20) the magnitude of the vector product is

$$AB \sin\phi = (6)(4)(\sin 30°) = 12$$

By the right-hand rule, the direction of $\vec{A} \times \vec{B}$ is along the $+z$-axis (the direction of the unit vector $\hat{k}$), so $\vec{C} = \vec{A} \times \vec{B} = 12\hat{k}$.

To use Eqs. (1.25), we first determine the components of $\vec{A}$ and $\vec{B}$. Note that $\vec{A}$ points along the x-axis, so its only nonzero component is A_x. For $\vec{B}$, Fig. 1.33 shows that $\phi = 30°$ is measured from the $+x$-axis toward the $+y$-axis, so we can use Eqs. (1.5):

$$A_x = 6 \qquad A_y = 0 \qquad A_z = 0$$

$$B_x = 4\cos 30° = 2\sqrt{3} \qquad B_y = 4\sin 30° = 2 \qquad B_z = 0$$

Then Eqs. (1.25) yield

$$C_x = (0)(0) - (0)(2) = 0$$

$$C_y = (0)(2\sqrt{3}) - (6)(0) = 0$$

$$C_z = (6)(2) - (0)(2\sqrt{3}) = 12$$

Thus again we have $\vec{C} = 12\hat{k}$.

EVALUATE: Both methods give the same result. Depending on the situation, one or the other of the two approaches may be the more convenient one to use.

TEST YOUR UNDERSTANDING OF SECTION 1.10 Vector $\vec{A}$ has magnitude 2 and vector $\vec{B}$ has magnitude 3. The angle ϕ between $\vec{A}$ and $\vec{B}$ is (i) 0°, (ii) 90°, or (iii) 180°. For each of the following situations, state what the value of ϕ must be. (In each situation there may be more than one correct answer.) (a) $\vec{A} \cdot \vec{B} = 0$; (b) $\vec{A} \times \vec{B} = \mathbf{0}$; (c) $\vec{A} \cdot \vec{B} = 6$; (d) $\vec{A} \cdot \vec{B} = -6$; (e) (magnitude of $\vec{A} \times \vec{B}$) $= 6$.

CHAPTER 1 SUMMARY

SOLUTIONS TO ALL EXAMPLES

Physical quantities and units: Three fundamental physical quantities are mass, length, and time. The corresponding fundamental SI units are the kilogram, the meter, and the second. Derived units for other physical quantities are products or quotients of the basic units. Equations must be dimensionally consistent; two terms can be added only when they have the same units. (See Examples 1.1 and 1.2.)

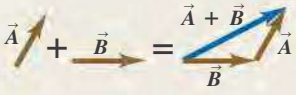
Significant figures: The accuracy of a measurement can be indicated by the number of significant figures or by a stated uncertainty. The significant figures in the result of a calculation are determined by the rules summarized in Table 1.2. When only crude estimates are available for input data, we can often make useful order-of-magnitude estimates. (See Examples 1.3 and 1.4.)

Significant figures in magenta

$$\pi = \frac{C}{2r} = \frac{0.424\ \text{m}}{2(0.06750\ \text{m})} = 3.14$$

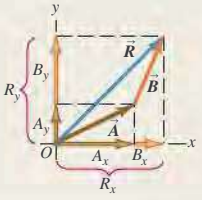
$$123.62 + 8.9 = 132.5$$

Scalars, vectors, and vector addition: Scalar quantities are numbers and combine according to the usual rules of arithmetic. Vector quantities have direction as well as magnitude and combine according to the rules of vector addition. The negative of a vector has the same magnitude but points in the opposite direction. (See Example 1.5.)



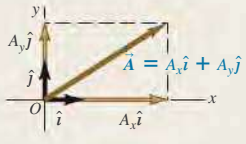
Vector components and vector addition: Vectors can be added by using components of vectors. The x-component of $\vec{R} = \vec{A} + \vec{B}$ is the sum of the x-components of $\vec{A}$ and $\vec{B}$, and likewise for the y- and z-components. (See Examples 1.6 and 1.7.)

$$R_x = A_x + B_x$$
$$R_y = A_y + B_y \qquad (1.9)$$
$$R_z = A_z + B_z$$



Unit vectors: Unit vectors describe directions in space. A unit vector has a magnitude of 1, with no units. The unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$, aligned with the x-, y-, and z-axes of a rectangular coordinate system, are especially useful. (See Example 1.8.)

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \qquad (1.14)$$



Scalar product: The scalar product $C = \vec{A} \cdot \vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ is a scalar quantity. It can be expressed in terms of the magnitudes of $\vec{A}$ and $\vec{B}$ and the angle ϕ between the two vectors, or in terms of the components of $\vec{A}$ and $\vec{B}$. The scalar product is commutative; $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$. The scalar product of two perpendicular vectors is zero. (See Examples 1.9 and 1.10.)

$$\vec{A} \cdot \vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi \qquad (1.16)$$
$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z \qquad (1.19)$$

Scalar product $\vec{A} \cdot \vec{B} = AB\cos\phi$



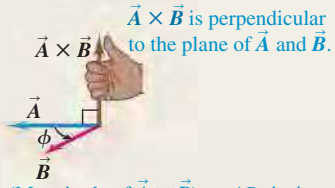
Vector product: The vector product $\vec{C} = \vec{A} \times \vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ is a third vector $\vec{C}$. The magnitude of $\vec{A} \times \vec{B}$ depends on the magnitudes of $\vec{A}$ and $\vec{B}$ and the angle ϕ between the two vectors. The direction of $\vec{A} \times \vec{B}$ is perpendicular to the plane of the two vectors being multiplied, as given by the right-hand rule. The components of $\vec{C} = \vec{A} \times \vec{B}$ can be expressed in terms of the components of $\vec{A}$ and $\vec{B}$. The vector product is not commutative; $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$. The vector product of two parallel or antiparallel vectors is zero. (See Example 1.11.)

$$C = AB\sin\phi \tag{1.20}$$

$$C_x = A_yB_z - A_zB_y$$

$$C_y = A_zB_x - A_xB_z \tag{1.25}$$

$$C_z = A_xB_y - A_yB_x$$

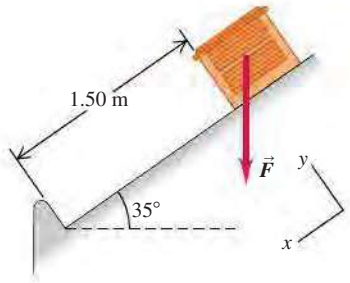


BRIDGING PROBLEM VECTORS ON THE ROOF

SOLUTION

An air-conditioning unit is fastened to a roof that slopes at an angle of 35° above the horizontal (**Fig. 1.34**). Its weight is a force $\vec{F}$ on the air conditioner that is directed vertically downward. In order that the unit not crush the roof tiles, the component of the unit's weight perpendicular to the roof cannot exceed 425 N. (One newton, or 1 N, is the SI unit of force. It is equal to 0.2248 lb.) (a) What is the maximum allowed weight of the unit? (b) If the fasteners fail, the unit slides 1.50 m along the roof before it comes to a halt against a ledge. How much work does the weight force do on the unit during its slide if the unit has the weight calculated in part (a)? The work done by a force $\vec{F}$ on an object that undergoes a displacement $\vec{s}$ is $W = \vec{F} \cdot \vec{s}$.

1.34 An air-conditioning unit on a slanted roof.



SOLUTION GUIDE

IDENTIFY and SET UP

1. This problem involves vectors and components. What are the known quantities? Which aspect(s) of the weight vector (magnitude, direction, and/or particular components) represent the target variable for part (a)? Which aspect(s) must you know to solve part (b)?
2. Make a sketch based on Fig. 1.34. Draw the x- and y-axes, choosing the positive direction for each. Your axes don't have to be horizontal and vertical, but they do have to be mutually perpendicular. Figure 1.34 shows a convenient choice of axes: The x-axis is parallel to the slope of the roof.
3. Choose the equations you'll use to determine the target variables.

EXECUTE

4. Use the relationship between the magnitude and direction of a vector and its components to solve for the target variable in part (a). Be careful: Is 35° the correct angle to use in the equation? (*Hint:* Check your sketch.)
5. Make sure your answer has the correct number of significant figures.
6. Use the definition of the scalar product to solve for the target variable in part (b). Again, use the correct number of significant figures.

EVALUATE

7. Did your answer to part (a) include a vector component whose absolute value is greater than the magnitude of the vector? Is that possible?
8. There are two ways to find the scalar product of two vectors, one of which you used to solve part (b). Check your answer by repeating the calculation, using the other way. Do you get the same answer?

Chapter 2

Motion Along a Straight Line

Goals for Chapter 2

- To describe straight-line motion in terms of velocity and acceleration
- To distinguish between average and instantaneous velocity and average and instantaneous acceleration
- To understand straight-line motion with constant acceleration
- To examine freely falling bodies

Introduction

- *Kinematics* is the study of motion.
- *Velocity* and *acceleration* are important physical quantities.

there are several KEY VARIABLES that we use.

Symbol	*Variable*	*Units*
T	***Time***	***s***
A	***Acceleration***	***m/s^2***
x or y	***Displacement***	***m***
v_o	***Initial velocity***	***m/s***
V	***Final velocity***	***m/s***
g or a_g	***Acceleration due to gravity***	***m/s^2***

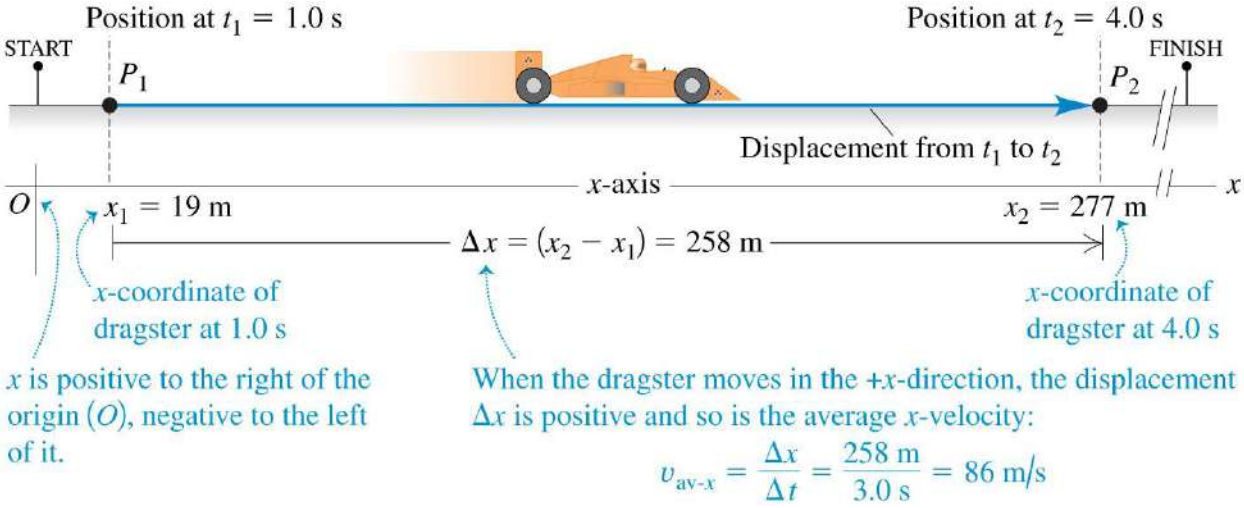
Displacement, time, and average velocity

- A particle moving along the x-axis has a coordinate x.
- The change in the particle's coordinate is $\Delta x = x_2 - x_1$.

$$\text{average velocity} = \frac{\text{displacement}}{\text{total travel time}}$$

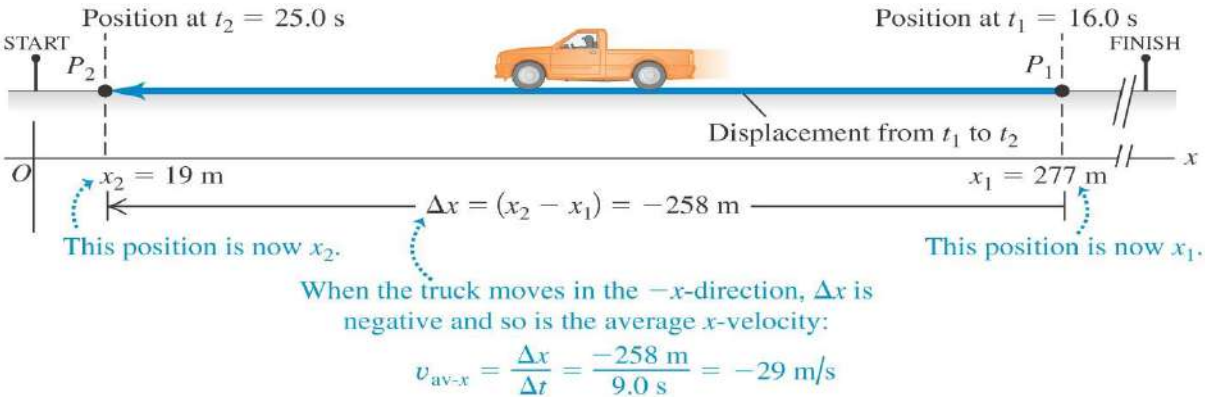
$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

- Figure 2.1 illustrates how these quantities are related.



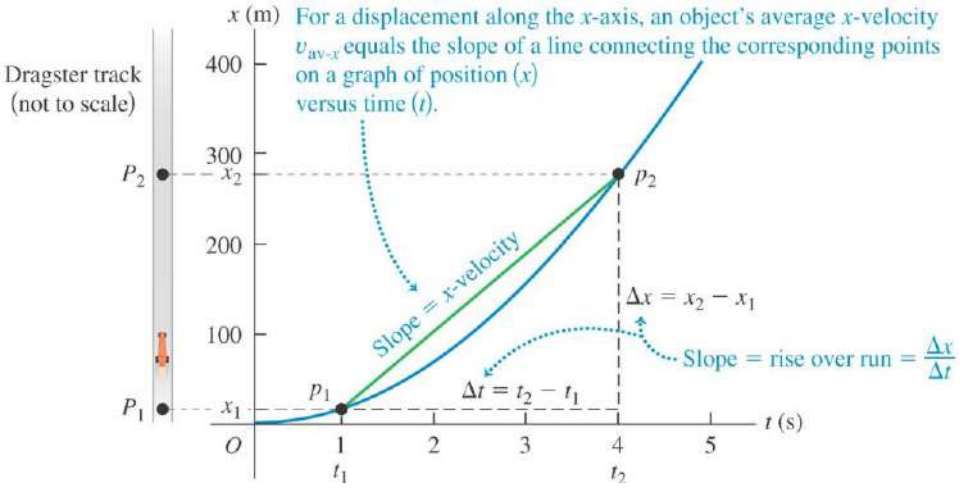
Negative velocity

- The average x-velocity is *negative* during a time interval if the particle moves in the negative x-direction for that time interval. Figure 2.2 illustrates this situation.



A position-time graph

- A position-time graph (an *x*-*t* graph) shows the particle's position *x* as a function of time *t*.
- Figure 2.3 shows how the average *x*-velocity is related to the slope of an *x*-*t* graph.



Instantaneous velocity

- The instantaneous velocity is the instantaneous rate of change of position vector with respect to time.

The **instantaneous *x*-velocity** of a particle in **straight-line motion** ...

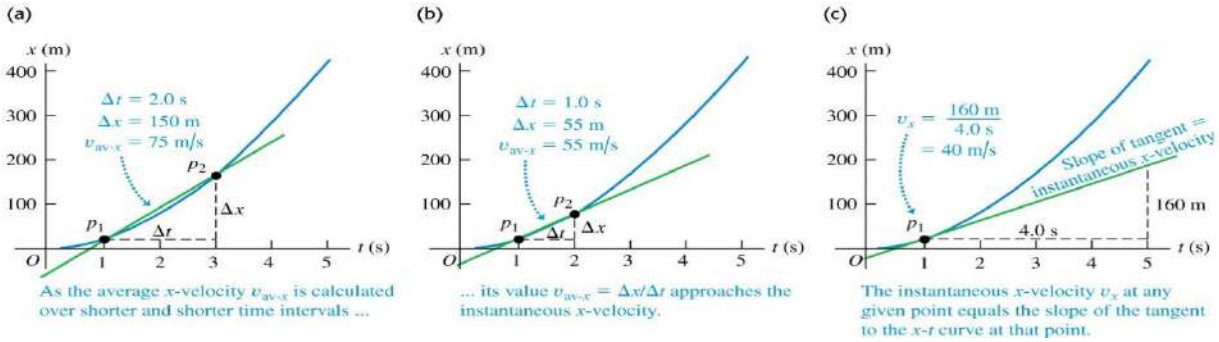
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (2.3)$$

... equals the limit of the particle's average *x*-velocity as the time interval approaches zero ...

... and equals the instantaneous rate of change of the particle's *x*-coordinate.

Finding velocity on an x-t graph

- At any point on an x-t graph, the instantaneous x-velocity is equal to the slope of the tangent to the curve at that point.



Average acceleration

- Acceleration describes the rate of change of velocity with time.

Average *x*-acceleration of a particle in **straight-line motion** during time interval from t_1 to t_2

$$a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{2x} - v_{1x}}{t_2 - t_1}$$

Change in *x*-component of the particle's velocity

Final *x*-velocity minus initial *x*-velocity

Time interval

Final time minus initial time

Instantaneous acceleration

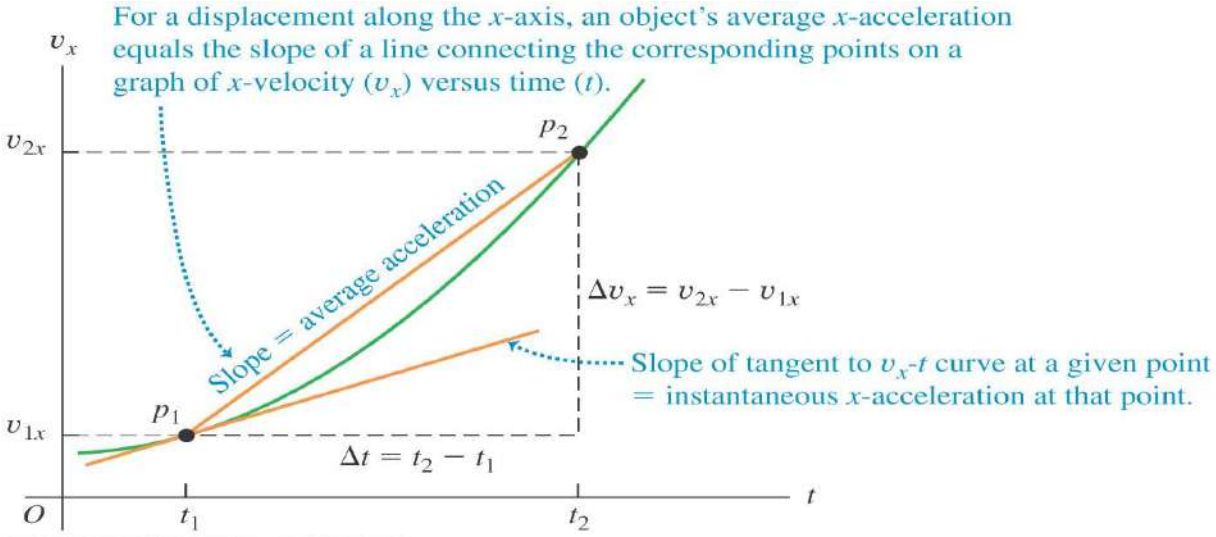
- The instantaneous acceleration is the instantaneous rate of change of the velocity with respect to time.

The **instantaneous *x*-acceleration** of a particle in **straight-line motion** ...

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

... equals the limit of the particle's average *x*-acceleration as the time interval approaches zero ...

... and equals the instantaneous rate of change of the particle's *x*-velocity.



As shown in Figure, the υ_x-t graph may be used to find the instantaneous acceleration and the average acceleration.

MOTION WITH CONSTANT ACCELERATION

If the acceleration is constant, we can find the velocity as a function of time

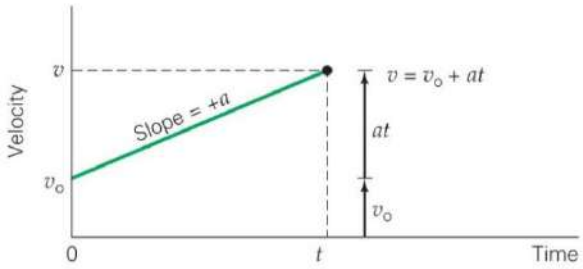
***x*-velocity** at time t of a particle with **constant *x*-acceleration**

$$v_x = v_{0x} + a_x t$$

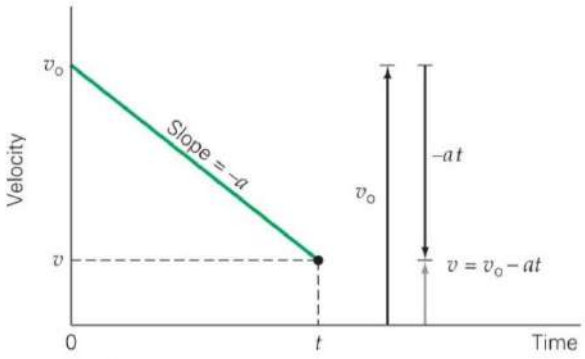
x-velocity of the particle at time 0

Constant *x*-acceleration of the particle

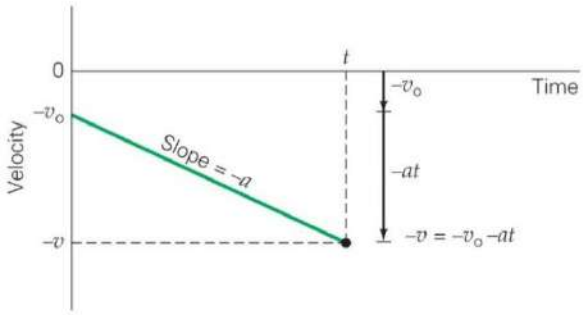
Time



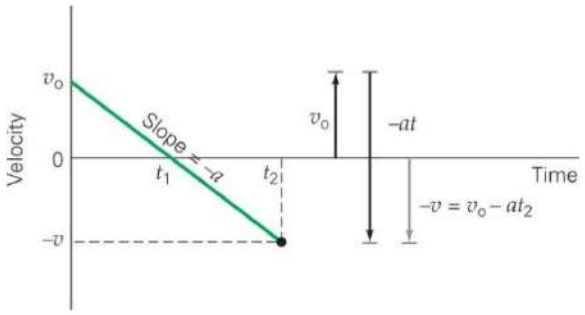
(a) Motion in positive direction—speeding up



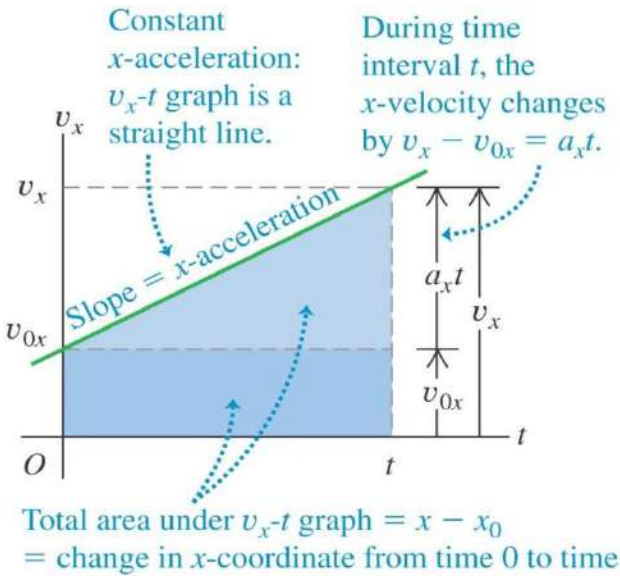
(b) Motion in positive direction—slowing down



(c) Motion in negative direction—speeding up



(d) Changing direction

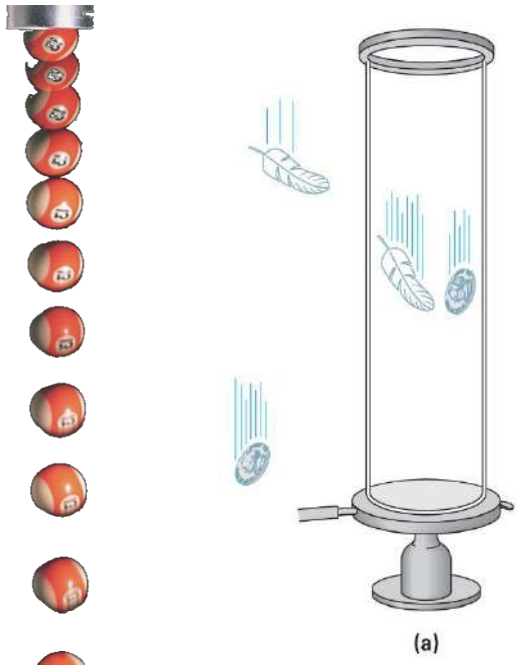


• The four equations shown apply to any straight-line motion with constant acceleration a_x.

KINEMATICS	v_i	v_f	Δt	a	Δx
$\Delta x = 1/2\,(v_i + v_f)\Delta t$	X	X	X	no	X
$v_f = v_i + a\Delta t$	X	X	X	X	no
$\Delta x = v_i\Delta t + 1/2 a\Delta t^2$	X	no	X	X	X
$v_f^2 = v_i^2 + 2a\Delta x$	X	X	no	X	X

Freely falling bodies

- Free fall is the motion of an object under the influence of only gravity.
- In the figure, a strobe light flashes with equal time intervals between flashes.
- The velocity change is the same in each time interval, so the acceleration is constant.
- Aristotle thought that heavy bodies fall faster than light ones, but Galileo showed that all bodies fall at the *same* rate.
- If there is no air resistance, the downward acceleration of any freely falling object is $g = 9.8\ \text{m/s}^2$.



$$v = v_o - gt$$
$$y = y_o + v_o t - \tfrac{1}{2} g t^2$$
$$v^2 = v_o^2 - 2g(y - y_o)$$
$$y = y_o + \tfrac{1}{2}(v + v_o)t$$

The positive *y*-direction has been chosen to be upwards. If it is chosen to be downwards, the sign of *g* would need to be changed.

Example: A boat moves slowly out of a marina (so as to not leave a wake) with a speed of 1.50 m/s. As soon as it passes the breakwater, leaving the marina, it throttles up and accelerates at 2.4 m/s^2.

a) How fast is the boat moving after accelerating for 5 seconds?
b) How far did the boat travel during that time?

Example: You are driving through town at 12 m/s when suddenly a ball rolls out in front of your car. You apply the brakes and begin decelerating at 3.5 m/s2.

How far do you travel before coming to a complete stop?

Example: A car accelerates from 12.5 m/s to 25 m/s in 6 seconds. What was the acceleration?

Example: A pitcher throws a fastball with a velocity of 43.5 m/s. It is determined that during the windup and delivery the ball covers a displacement of 2.5 meters. This is from the point behind the body when the ball is at rest to the point of release. Calculate the acceleration during his throwing motion.

Example: How long does it take a car at rest to cross a 35 m intersection after the light turns green, if the acceleration of the car is a constant 2 m/s^2?

Example: A stone is dropped at rest from the top of a cliff. It is observed to hit the ground 5.78 s later. How high is the cliff?

Motion in Two Dimensions

- Reminder of vectors and vector algebra
- Displacement and position in 2-D
- Average and instantaneous velocity in 2-D
- Average and instantaneous acceleration in 2-D
- Projectile motion
- Uniform circular motion

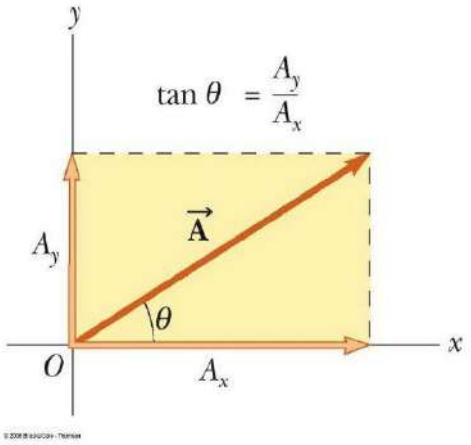
Vector and its components

- The components are the legs of the right triangle whose hypotenuse is *A*

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

$$\begin{cases} A_x = A\cos(\theta) \\ A_y = A\sin(\theta) \end{cases}$$

$$\begin{cases} |\vec{A}| = \sqrt{(A_x)^2 + (A_y)^2} \\ \tan(\theta) = \dfrac{A_y}{A_x} \quad \text{or} \quad \theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right) \end{cases}$$

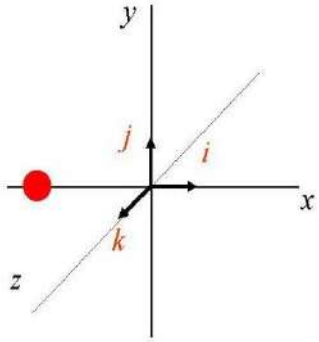


Motion in two dimensions

- Kinematic variables in one dimension
  - Position: $x(t)$ m
  - Velocity: $v(t)$ m/s
  - Acceleration: $a(t)$ m/s²



- Kinematic variables in three dimensions
  - Position: $\vec{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$ m
  - Velocity: $\vec{v}(t) = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$ m/s
  - Acceleration: $\vec{a}(t) = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ m/s²



- All are vectors: have direction and magnitudes

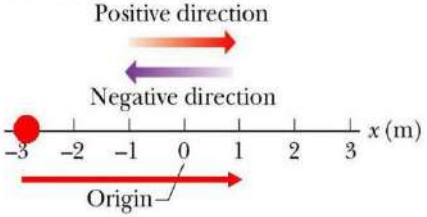
Position and Displacement

- In one dimension

$$\Delta x = x_2(t_2) - x_1(t_1)$$

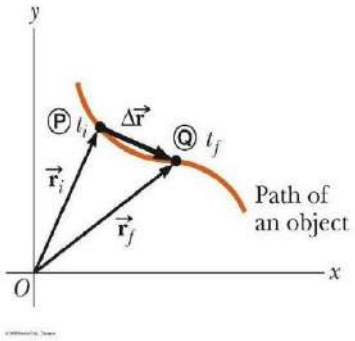
$x_1(t_1) = -3.0$ m, $x_2(t_2) = +1.0$ m

$\Delta x = +1.0\text{ m} + 3.0\text{ m} = +4.0$ m



- In two dimensions
  - Position: the position of an object is described by its position vector $\vec{r}(t)$ --always points to particle from origin.
  - Displacement: $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$$\Delta\vec{r} = (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})$$
$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$
$$= \Delta x\hat{i} + \Delta y\hat{j}$$



Average & Instantaneous Velocity

- Average velocity $\vec{v}_{avg} \equiv \dfrac{\Delta\vec{r}}{\Delta t}$

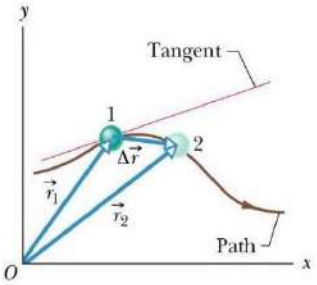
$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = v_{avg,x}\hat{i} + v_{avg,y}\hat{j}$$

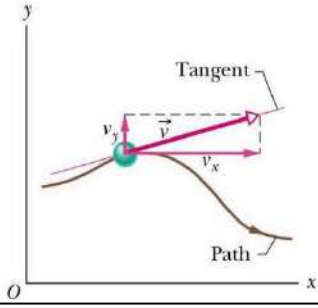
- Instantaneous velocity

$$\vec{v} \equiv \lim_{t\to 0}\vec{v}_{avg} = \lim_{t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

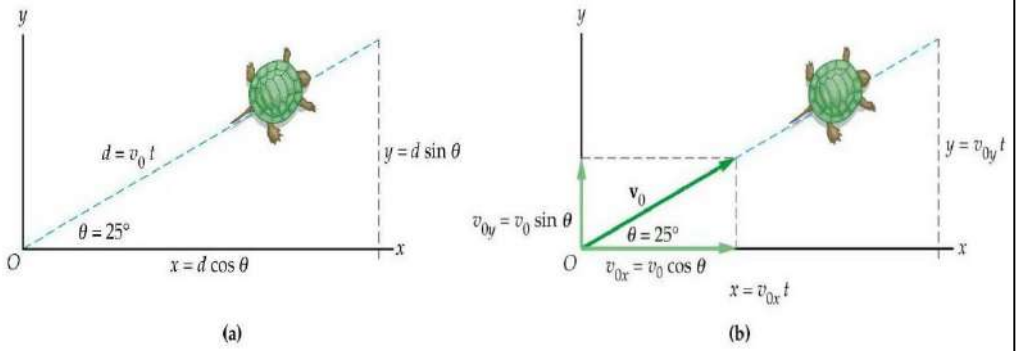
$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

- v is tangent to the path in x-y graph;





Motion of a Turtle



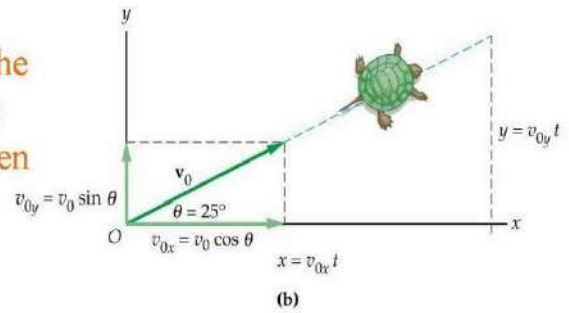
A turtle starts at the origin and moves with the speed of v_0=10 cm/s in the direction of 25° to the horizontal.

(a) Find the coordinates of a turtle 10 seconds later.

(b) How far did the turtle walk in 10 seconds?

Motion of a Turtle

Notice, you can solve the equations independently for the horizontal (x) and vertical (y) components of motion and then combine them!



$$\vec{v}_0 = \vec{v}_x + \vec{v}_y$$

- X components:

$$v_{0x} = v_0 \cos 25^\circ = 9.06 \text{ cm/s} \qquad \Delta x = v_{0x} t = 90.6 \text{ cm}$$

- Y components:

$$v_{0y} = v_0 \sin 25^\circ = 4.23 \text{ cm/s} \qquad \Delta y = v_{0y} t = 42.3 \text{ cm}$$

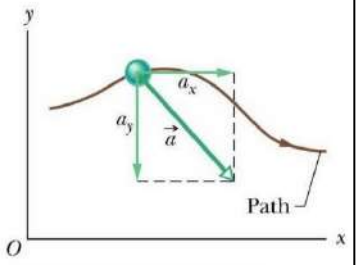
- Distance from the origin:

$$d = \sqrt{\Delta x^2 + \Delta y^2} = 100.0 \text{ cm}$$

Average & Instantaneous Acceleration

- Average acceleration $\vec{a}_{avg} \equiv \dfrac{\Delta \vec{v}}{\Delta t}$

$$\vec{a}_{avg} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} = a_{avg,x}\hat{i} + a_{avg,y}\hat{j}$$



- Instantaneous acceleration

$$\vec{a} \equiv \lim_{t\to 0} \vec{a}_{avg} = \lim_{t\to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

- The magnitude of the velocity (the speed) can change
- The direction of the velocity can change, even though the magnitude is constant
- Both the magnitude and the direction can change

Summary in two dimension

- Position $\vec{r}(t) = x\hat{i} + y\hat{j}$
- Average velocity $\vec{v}_{avg} = \dfrac{\Delta \vec{r}}{\Delta t} = \dfrac{\Delta x}{\Delta t}\hat{i} + \dfrac{\Delta y}{\Delta t}\hat{j} = v_{avg,x}\hat{i} + v_{avg,y}\hat{j}$
- Instantaneous velocity $v_x \equiv \dfrac{dx}{dt}$ $v_y \equiv \dfrac{dy}{dt}$

$$\vec{v}(t) = \lim_{t\to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

- Acceleration $a_x \equiv \dfrac{dv_x}{dt} = \dfrac{d^2x}{dt^2}$ $a_y \equiv \dfrac{dv_y}{dt} = \dfrac{d^2y}{dt^2}$

$$\vec{a}(t) = \lim_{t\to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

- $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ are not necessarily same direction.

Motion in two dimensions

- Motions in each dimension are independent components
- Constant acceleration equations

$$\vec{v} = \vec{v}_0 + \vec{a}t \qquad \vec{r} - \vec{r} = \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

- Constant acceleration equations hold in each dimension

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

$$x - x_0 = v_{0x} t + \tfrac{1}{2} a_x t^2 \qquad y - y_0 = v_{0y} t + \tfrac{1}{2} a_y t^2$$

$$v_x^{\,2} = v_{0x}^{\,2} + 2a_x(x - x_0) \qquad v_y^{\,2} = v_{0y}^{\,2} + 2a_y(y - y_0)$$

- $t = 0$ beginning of the process;
- $\vec{a} = a_x \hat{i} + a_y \hat{j}$ where a_x and a_y are constant;
- Initial velocity $\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j}$ initial displacement $\vec{r}_0 = x_0\hat{i} + y_0\hat{j}$

Hints for solving problems

- Define coordinate system. Make sketch showing axes, origin.
- List known quantities. Find v_{0x}, v_{0y}, a_x, a_y, etc. Show initial conditions on sketch.
- List equations of motion to see which ones to use.
- Time t is the same for x and y directions.

  $x_0 = x(t = 0)$, $y_0 = y(t = 0)$, $v_{0x} = v_x(t = 0)$, $v_{0y} = v_y(t = 0)$.
- Have an axis point along the direction of a if it is constant.

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

$$x - x_0 = v_{0x} t + \tfrac{1}{2} a_x t^2 \qquad y - y_0 = v_{0y} t + \tfrac{1}{2} a_y t^2$$

$$v_x^{\,2} = v_{0x}^{\,2} + 2a_x(x - x_0) \qquad v_y^{\,2} = v_{0y}^{\,2} + 2a_y(y - y_0)$$

Projectile Motion

- 2-D problem and define a coordinate system: x- horizontal, y- vertical (up +)
- Try to pick $x_0 = 0, y_0 = 0$ at $t = 0$
- Horizontal motion + Vertical motion
- Horizontal: $a_x = 0$, constant velocity motion
- Vertical: $a_y = -g = -9.8\ \text{m/s}^2$, $v_{0y} = 0$
- Equations:

Horizontal

$$v_x = v_{0x} + a_x t$$

$$x - x_0 = v_{0x} t + \tfrac{1}{2} a_x t^2$$

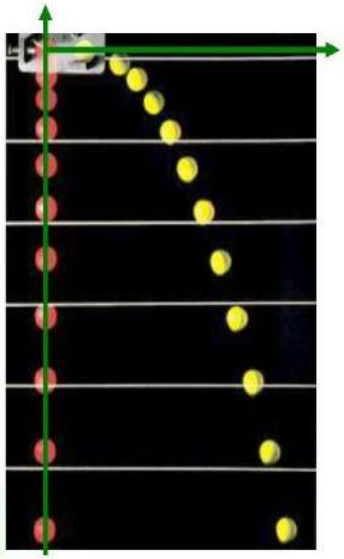
$$v_x^{\,2} = v_{0x}^{\,2} + 2a_x(x - x_0)$$

Vertical

$$v_y = v_{0y} + a_y t$$

$$y - y_0 = v_{0y} t + \tfrac{1}{2} a_y t^2$$

$$v_y^{\,2} = v_{0y}^{\,2} + 2a_y(y - y_0)$$



Projectile Motion

- X and Y motions happen independently, so we can treat them separately

Horizontal

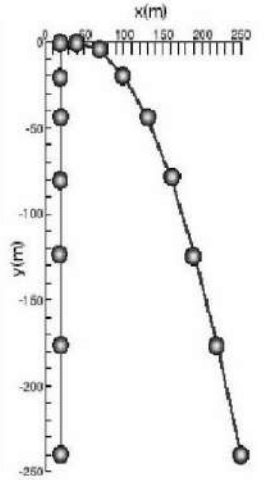
$$v_x = v_{0x}$$

$$x = x_0 + v_{0x} t$$

Vertical

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y} t - \tfrac{1}{2} g t^2$$



- Try to pick $x_0 = 0, y_0 = 0$ at $t = 0$
- Horizontal motion + Vertical motion
- Horizontal: $a_x = 0$, constant velocity motion
- Vertical: $a_y = -g = -9.8\ \text{m/s}^2$
- x and y are connected by time t
- y(x) is a parabola

Projectile Motion



- 2-D problem and define a coordinate system.
- Horizontal: $a_x = 0$ and vertical: $a_y = -g$.
- Try to pick $x_0 = 0, y_0 = 0$ at $t = 0$.
- Velocity initial conditions:
  - v_0 can have x, y components.
  - v_{0x} is constant usually. $v_{0x} = v_0 \cos\theta_0$
  - v_{0y} changes continuously. $v_{0x} = v_0 \sin\theta_0$
- Equations:

Horizontal	Vertical
$v_x = v_{0x}$	$v_y = v_{0y} - gt$
$x = x_0 + v_{0x}t$	$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

Trajectory of Projectile Motion

- Initial conditions ($t = 0$): $x_0 = 0, y_0 = 0$
  $v_{0x} = v_0 \cos\theta_0$ and $v_{0y} = v_0 \sin\theta_0$
- Horizontal motion:

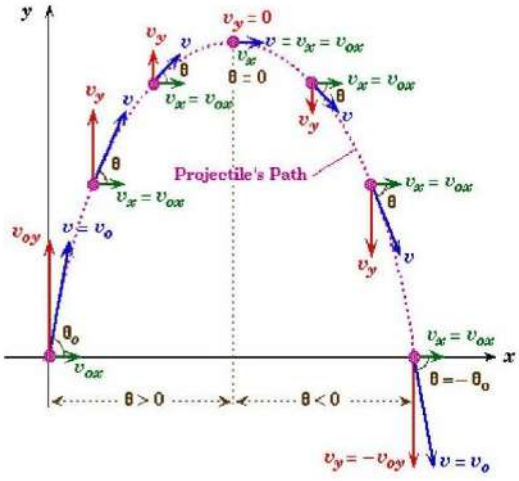
$$x = 0 + v_{0x}t \quad \Rightarrow \quad t = \frac{x}{v_{0x}}$$

- Vertical motion:

$$y = 0 + v_{0y}t - \frac{1}{2}gt^2$$

$$y = v_{0y}\left(\frac{x}{v_{0x}}\right) - \frac{g}{2}\left(\frac{x}{v_{0x}}\right)^2$$

$$y = x\tan\theta_0 - \frac{g}{2v_0^2\cos^2\theta_0}x^2$$



- Parabola;
  - $\theta_0 = 0$ and $\theta_0 = 90$?

What is R and h ?

- Initial conditions ($t = 0$): $x_0 = 0, y_0 = 0$
  $v_{0x} = v_0 \cos\theta_0$ and $v_{0x} = v_0 \sin\theta_0$, then

$$x = 0 + v_{0x}t \qquad 0 = 0 + v_{0y}t - \frac{1}{2}gt^2$$

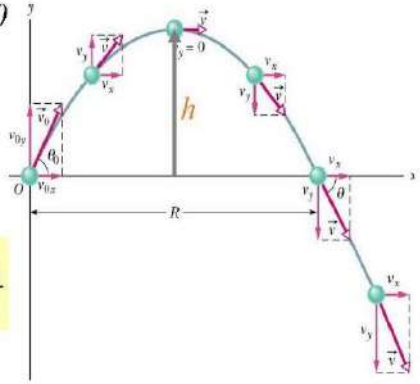
$$t = \frac{2v_{0y}}{g} = \frac{2v_0\sin\theta_0}{g}$$

$$R = x - x_0 = v_{0x}t = \frac{2v_0\cos\theta_0 v_0\sin\theta_0}{g} = \frac{v_0^2\sin 2\theta_0}{g}$$

$$h = y - y_0 = v_{0y}t_h - \frac{1}{2}gt_h^2 = v_{0y}\frac{t}{2} - \frac{g}{2}\left(\frac{t}{2}\right)^2$$

$$h = \frac{v_0^2\sin^2\theta_0}{2g}$$

$$v_y = v_{0y} - gt = v_{0y} - g\frac{2v_{0y}}{g} = -v_{0y}$$

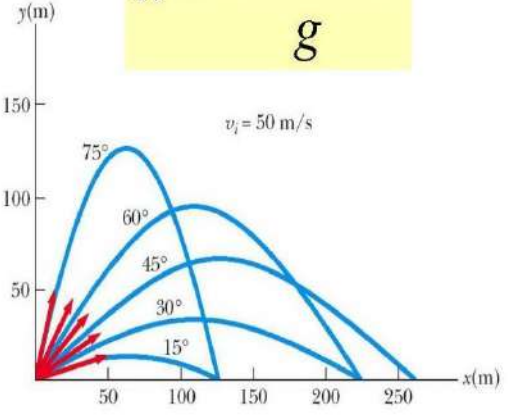


Horizontal	Vertical
$v_x = v_{0x}$	$v_y = v_{0y} - gt$
$x = x_0 + v_{0x}t$	$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

Projectile Motion at Various Initial Angles

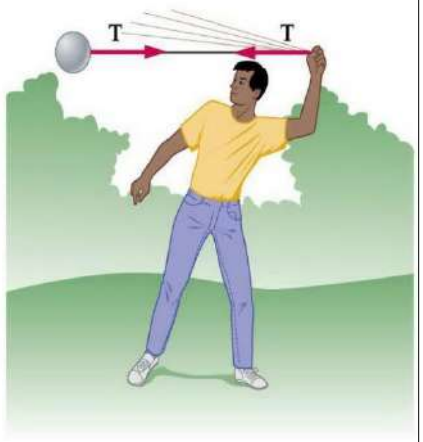
- Complementary values of the initial angle result in the same range
  - The heights will be different
- The maximum range occurs at a projection angle of 45°

$$R = \frac{v_0^2\sin 2\phi}{g}$$



Circular Motion: Observations

- Object moving along a curved path with constant speed
  - Magnitude of velocity: same
  - Direction of velocity: changing
  - Velocity: changing
  - Acceleration is NOT zero!
  - Net force acting on the object is NOT zero
  - "Centripetal force"

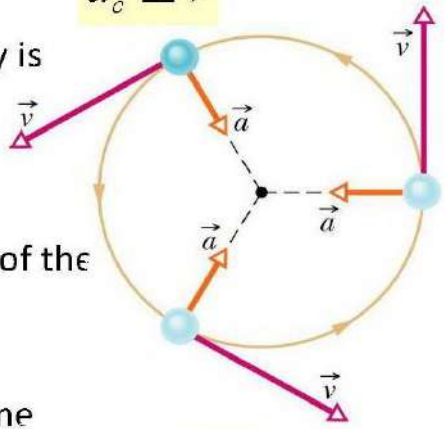


$$\vec{F}_{net} = m\vec{a}$$

Uniform Circular Motion

- Velocity:
  - Magnitude: constant v
  - The direction of the velocity is tangent to the circle
- Acceleration:
  - Magnitude: $a_c = \frac{v^2}{r}$
  - directed toward the center of the circle of motion
- Period:
  - time interval required for one complete revolution of the particle $T = \frac{2\pi r}{v}$

$\vec{a}_c \perp \vec{v}$



Summary

- Position $\vec{r}(t) = x\hat{i} + y\hat{j}$
- Average velocity $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = v_{avg,x}\hat{i} + v_{avg,y}\hat{j}$
- Instantaneous velocity $v_x \equiv \frac{dx}{dt}$ $v_y \equiv \frac{dy}{dt}$

$$\vec{v}(t) = \lim_{t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

- Acceleration $a_x \equiv \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$ $a_y \equiv \frac{dv_y}{dt} = \frac{d^2y}{dt^2}$

$$\vec{a}(t) = \lim_{t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

- $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ are not necessarily in the same direction.

Summary

- If a particle moves with constant acceleration a, motion equations are

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2$$

$$\vec{r}_f = x_f\hat{i} + y_f\hat{j} = (x_i + v_{xi}t + \tfrac{1}{2}a_{xi}t^2)\hat{i} + (y_i + v_{yi}t + \tfrac{1}{2}a_{yi}t^2)\hat{j}$$

$$\vec{v} = \vec{v}_i + \vec{a}t$$

$$\vec{v}_f(t) = v_{fx}\hat{i} + v_{fy}\hat{j} = (v_{ix} + a_x t)\hat{i} + (v_{iy} + a_y t)\hat{j}$$

- Projectile motion is one type of 2-D motion under constant acceleration, where $a_x = 0$, $a_y = -g$.

A 600 N object is to be given an acceleration of 0.5 m/s^2. How large an unbalanced force must act upon it?

W= 600 N, a=0.5 m/s^2 , F=?

$$F=m\times a$$

$$W=m\times g$$

$$m=\frac{W}{g}=\frac{600}{10}=60kg$$

F= 60 kg× 0.5 m/s^2= 30 N

#6: A tow rope will break if the tension in it exceeds 1400 N. It is used to tow a 700-kg car along level ground. What is the largest acceleration the rope can give to the car?

F=1400 N, m= 700 kg, m=?

$$F=m\times a$$

$$a=\frac{F}{m}=\frac{1400}{700}=2\ \mathrm{m}/s^2$$

#11: A 12.0-g bullet is accelerated from rest to a speed of 700 m/s as it travels 20 cm in a gun barrel. Assuming the acceleration to be constant, how large was the accelerating force?

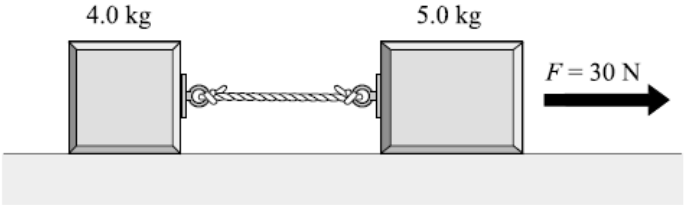
m=12.0 g=0.012 kg, v_1 =0, v_2 =700 m/s, , d=20 cm=0.20 m, *F=?*

$$v_2^2=v_1^2+2ax$$

$$a=\frac{v_2^2-v_1^2}{2x}=\frac{700^2-0}{2\times 0.2}=1225000\ \mathrm{m}/s^2$$

F=m×a=0.012 kg×1225000 m/s2=14700 N

#12: Find the acceleration of the blocks in Figure below:



m_1=4 kg, m_2=5kg, F=30N, *a=?*

F=m×a

$$a=\frac{F}{m}=\frac{30}{4+5}=3.33\ \mathrm{m}/s^2$$

The Laws of Motion

By

Dr. Ammar A. AlRawi

Mathematics and
Computer Applications Department
Collage of Science
2025-2026

The Laws of Motion

- Newton’s first law
- Force
- Mass
- Newton’s second law
- Newton’s third law
- Examples



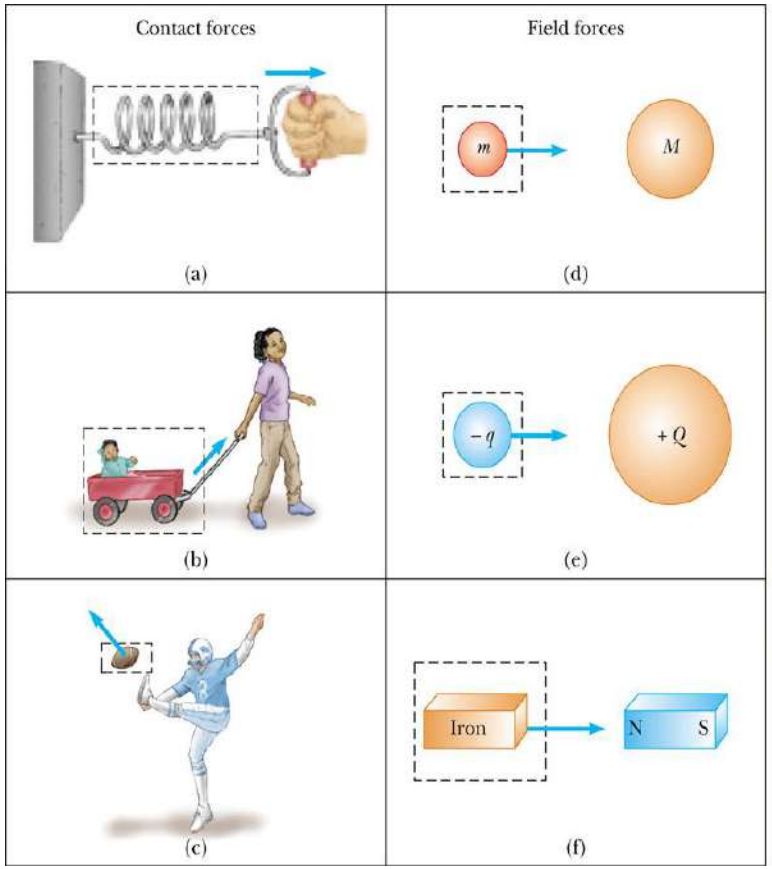
Isaac Newton’s work represents one of the greatest contributions to science ever made by an individual.

Dynamics

- Describes the relationship between the motion of objects in our everyday world and the forces acting on them
- Language of Dynamics
  - **Force:** The measure of interaction between **two objects** (pull or push). It is a vector quantity – it has a ***magnitude and direction***
  - **Mass:** The measure of how difficult it is to change object's velocity (sluggishness or inertia of the object)

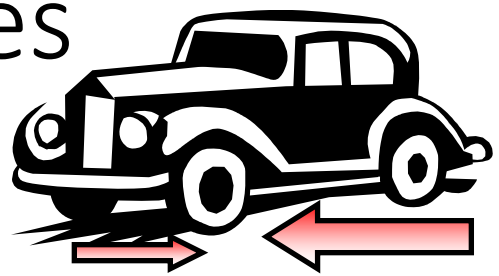
Forces

- The measure of interaction between **two objects** (pull or push)
- Vector quantity: has ***magnitude and direction***
- May be a **contact force** or a **field force**
  - Contact forces result from physical contact between two objects
  - Field forces act between disconnected objects
    - Also called “action at a distance”



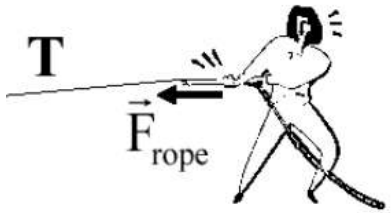
© 2002 Brooks Cole Publishing - a division of Thomson Learning

Forces

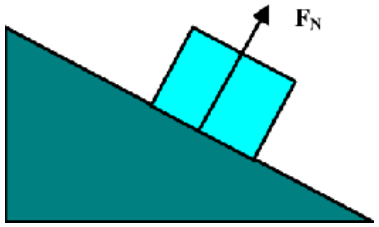


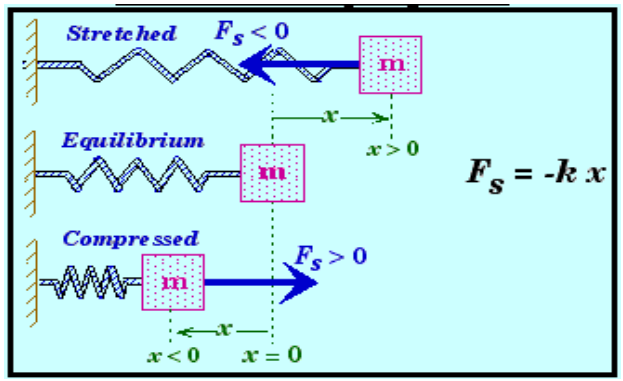


- Gravitational Force
- Archimedes Force
- Friction Force
- Tension Force
- Spring Force
- Normal Force







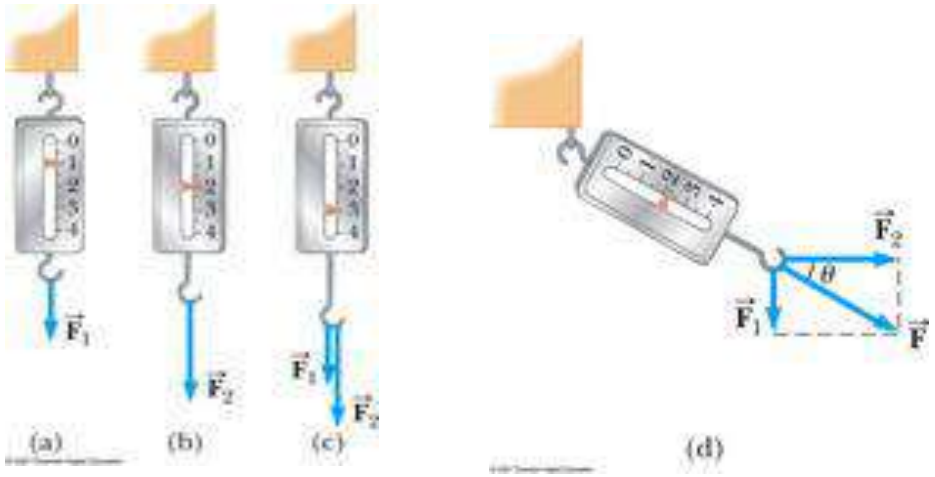


Vector Nature of Force

- Vector force: has ***magnitude and direction***
- Net Force: a resultant force acting on object

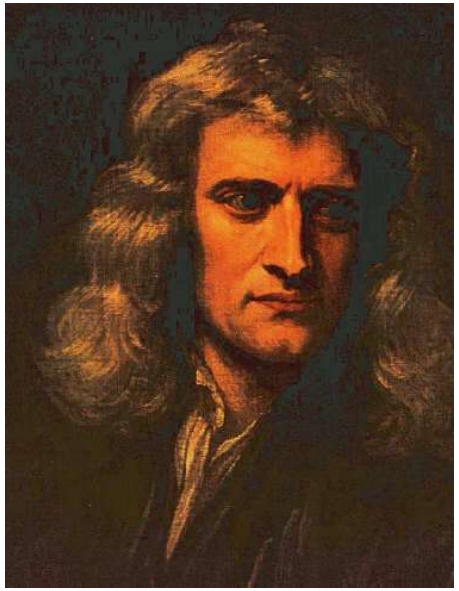
$$\vec{F}_{net} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots$$

- You must use the rules of vector addition to obtain the net force on an object



$$|\vec{F}| = \sqrt{F_1^2 + F_2^2} = 2.24\,\text{N}$$

$$\theta = \tan^{-1}\left(\frac{F_1}{F_2}\right) = -26.6^\circ$$

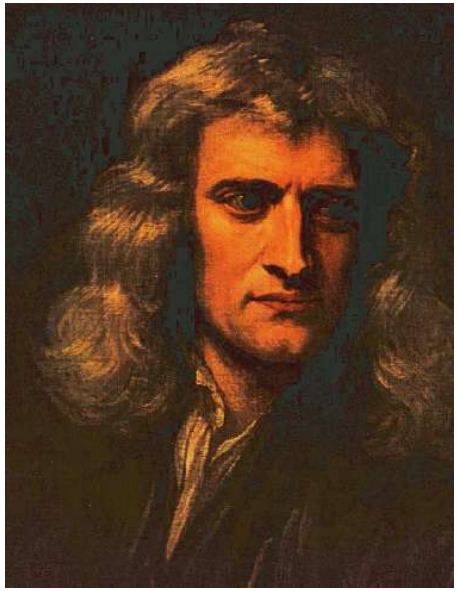


Newton's First Law

- An object at rest tends to stay at rest and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force

- An object at rest remains at rest as long as no **net force** acts on it
- An object moving with **constant velocity** continues to move with the same speed and in the same direction (the same velocity) as long as no net force acts on it
- *"Keep on doing what it is doing"*





Newton's First Law

- An object at rest tends to stay at rest and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force

- When forces are balanced, the acceleration of the object is zero
  - Object at rest: $v = 0$ and $a = 0$
  - Object in motion: $v \neq 0$ and $a = 0$

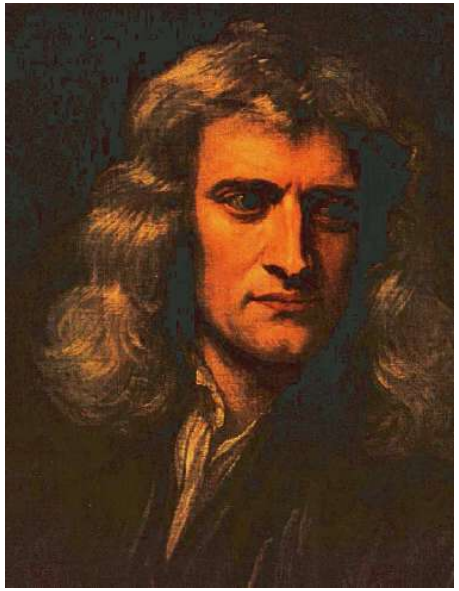
- The net force is defined as the vector sum of all the external forces exerted on the object. If the net force is zero, forces are balanced. When forces are balances, the object can be stationary, or move with constant velocity.

Mass and Inertia

- Every object continues in its state of rest, or uniform motion in a straight line, unless it is compelled to change that state by unbalanced forces impressed upon it
- **Inertia** is a property of objects to resist changes is motion!
- **Mass** is a measure of the amount of **inertia**.

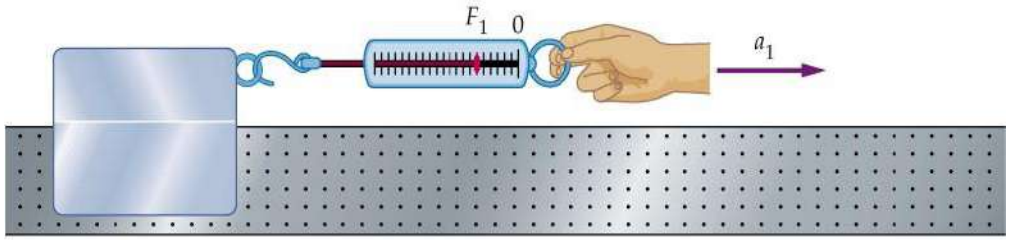


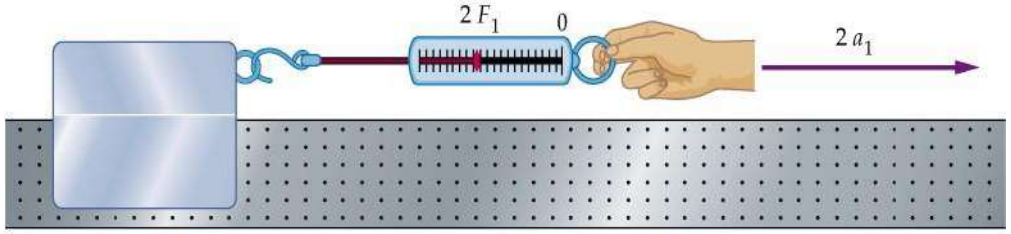
- **Mass** is a measure of the resistance of an object to changes in its velocity
- Mass is an inherent property of an object
- Scalar quantity and SI unit: kg



Newton's Second Law

- The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass





$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$\vec{F}_{net} = \sum \vec{F} = m\vec{a}$$

Units of Force

- Newton's second law:

$$\vec{F}_{net} = \sum \vec{F} = m\vec{a}$$

- SI unit of force is a Newton (N)

$$1\,\mathrm{N} \equiv 1\frac{\mathrm{kg\ m}}{\mathrm{s}^2}$$

More about Newton's 2nd Law

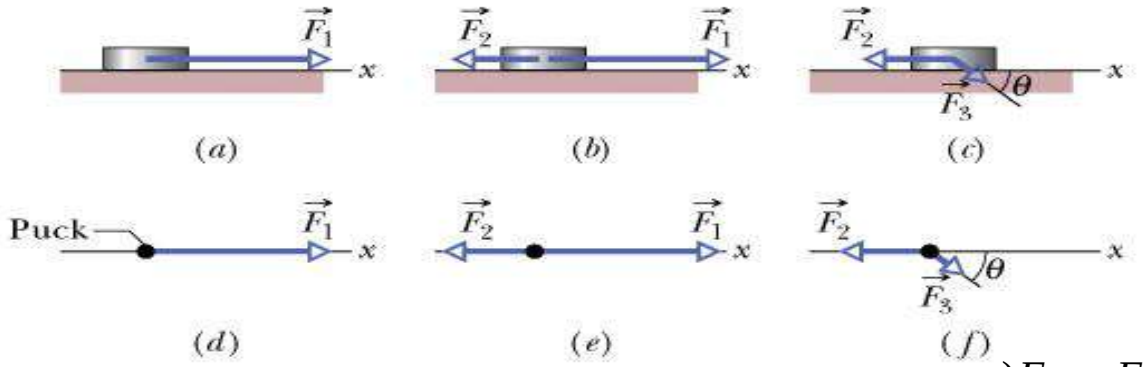
- You must be certain about which body we are applying it to
- $\mathbf{F}_{\text{net}}$ must be the vector sum of all the forces that act on that body
- Only forces that act on that body are to be included in the vector sum
- Net force component along an axis gives rise to the acceleration along that same axis

$$F_{net,x} = ma_x \qquad F_{net,y} = ma_y$$



Sample Problem

- One or two forces act on a puck that moves over frictionless ice along an x axis, in one-dimensional motion. The puck's mass is m = 0.20 kg. Forces $\mathbf{F_1}$ and $\mathbf{F_2}$ and are directed along the x axis and have magnitudes $F_1 = 4.0\ \mathrm{N}$ and $F_2 = 2.0\ \mathrm{N}$. Force $\mathbf{F_3}$ is directed at angle $\theta = 30°$ and has magnitude $F_3 = 1.0\ \mathrm{N}$. In each situation, what is the acceleration of the puck?



$$F_{net,x} = ma_x$$

$$a)\ F_1 = ma_x$$

$$a_x = \frac{F_1}{m} = \frac{4.0\ \mathrm{N}}{0.2\ \mathrm{kg}} = 20\ \mathrm{m/s^2}$$

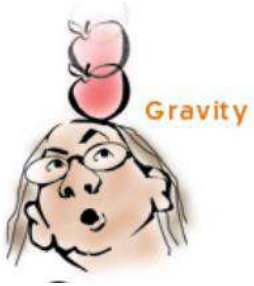
$$b)\ F_1 - F_2 = ma_x$$

$$a_x = \frac{F_1 - F_2}{m} = \frac{4.0\ \mathrm{N} - 2.0\ \mathrm{N}}{0.2\ \mathrm{kg}} = 10\ \mathrm{m/s^2}$$

$$c)\ F_{3,x} - F_2 = ma_x \qquad F_{3,x} = F_3 \cos\theta$$

$$a_x = \frac{F_3 \cos\theta - F_2}{m} = \frac{1.0\ \mathrm{N} \cos 30^\circ - 2.0\ \mathrm{N}}{0.2\ \mathrm{kg}} = -5.7\ \mathrm{m/s^2}$$

Gravitational Force



- Gravitational force is a vector
- Expressed by Newton's Law of Universal Gravitation:

$$F_g = G\frac{mM}{R^2}$$

  - G – gravitational constant
  - M – mass of the Earth
  - m – mass of an object
  - R – radius of the Earth
- Direction: pointing downward



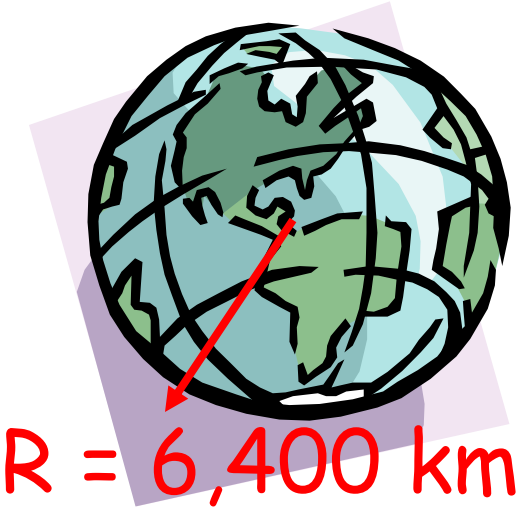
Weight

- The magnitude of the gravitational force acting on an object of mass m near the Earth's surface is called the weight w of the object: $w = mg$
- g can also be found from the Law of Universal Gravitation
- Weight has a unit of N

$$F_g = G\frac{mM}{R^2} \qquad w = F_g = mg$$

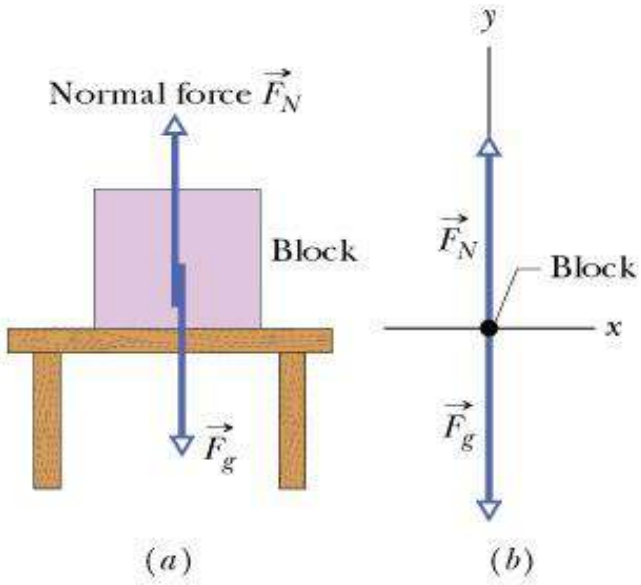
$$g = G\frac{M}{R^2} = 9.8\ \text{m/s}^2$$

- Weight depends upon location



Normal Force

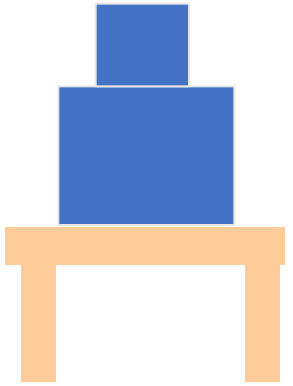
- Force from a solid surface which keeps object from falling through
- Direction: always perpendicular to the surface
- Magnitude: depends on situation



$$N - F_g = ma_y$$

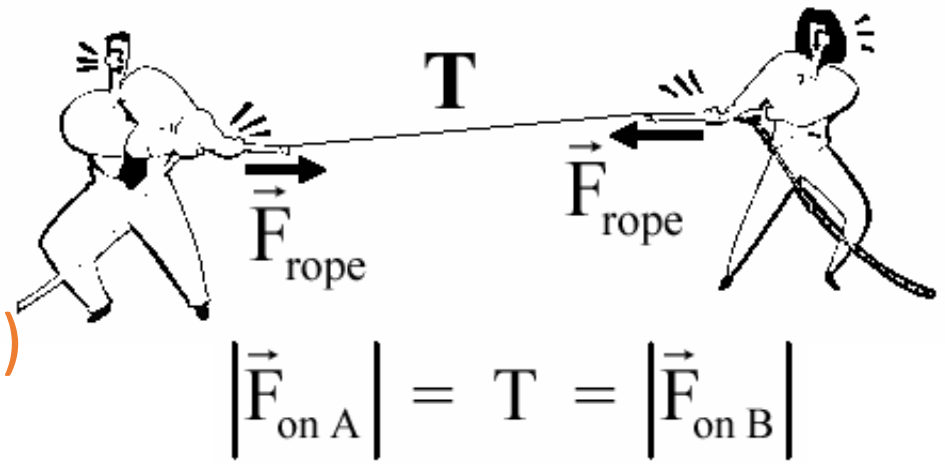
$$N - mg = ma_y$$

$$N = mg$$

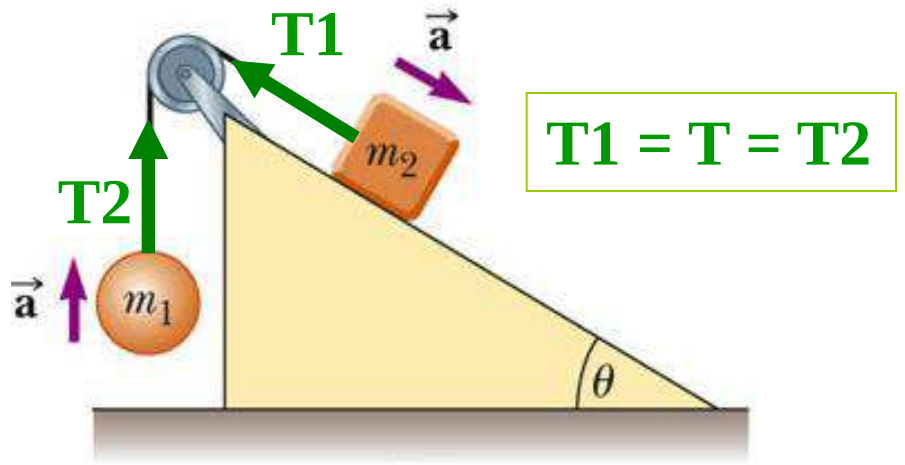


Tension Force: $\boldsymbol{T}$

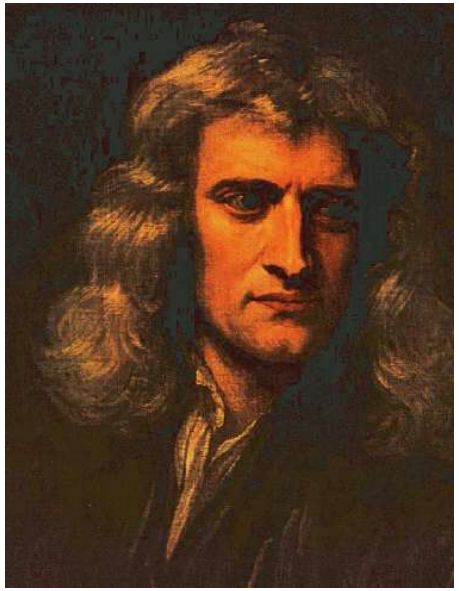
- A taut rope exerts forces on whatever holds its ends
- Direction: always along the cord (rope, cable, string ……) and away from the object
- Magnitude: depend on situation



$$\left|\vec{F}_{on\ A}\right| = T = \left|\vec{F}_{on\ B}\right|$$

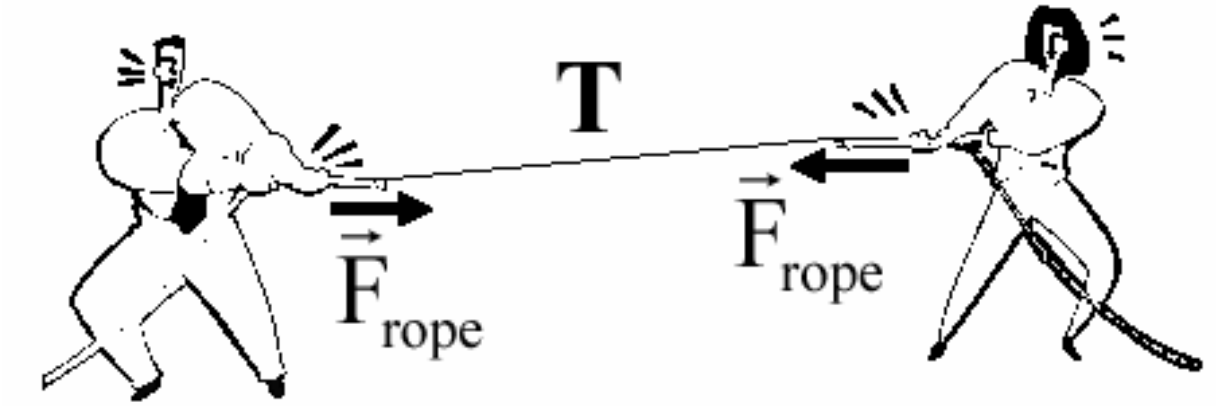


T1 = T = T2



Newton's Third Law

- If object 1 and object 2 interact, the force exerted by object 1 on object 2 is equal in magnitude but opposite in direction to the force exerted by object 2 on object 1



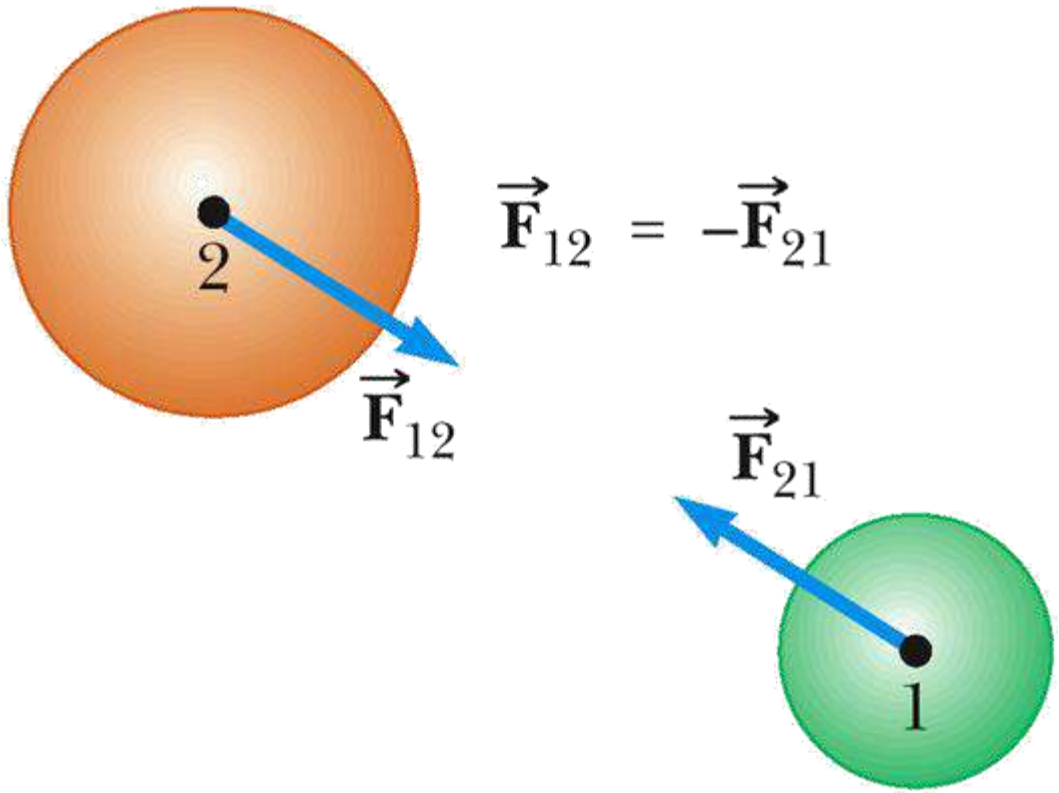
$$\left|\vec{F}_{\text{on A}}\right| = T = \left|\vec{F}_{\text{on B}}\right|$$

$$\vec{F}_{on\,A} = -\vec{F}_{on\,B}$$

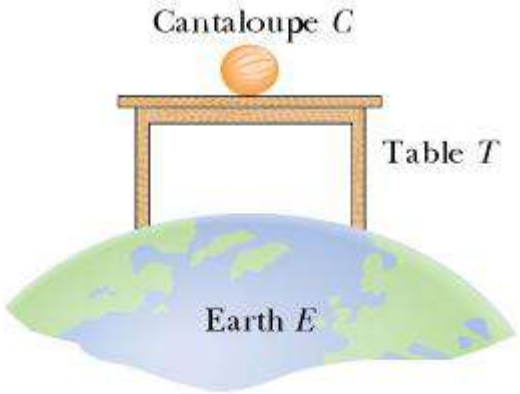
- Equivalent to saying a single isolated force cannot exist

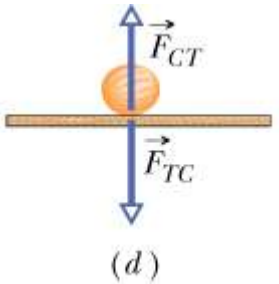
Newton's Third Law cont.

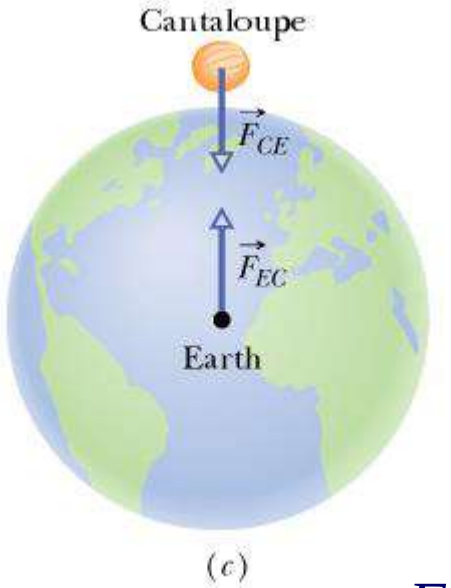
- F_{12} may be called the *action* force and F_{21} the *reaction* force
  - Actually, either force can be the action or the reaction force
- The action and reaction forces act on **different** objects



Some Action-Reaction Pairs









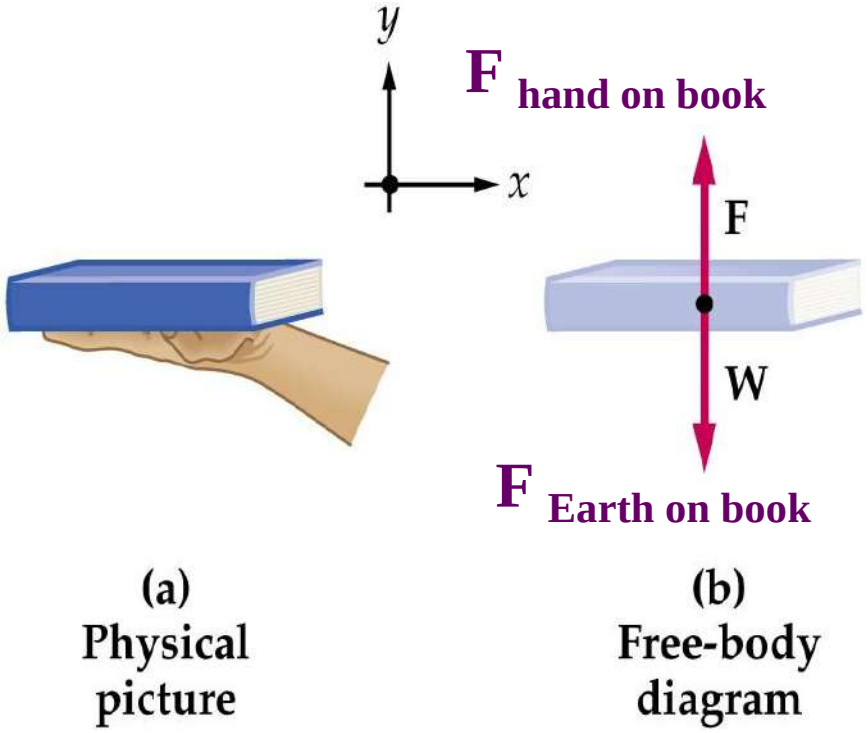
$$F_g = G\frac{mM}{R^2}$$

$$F_g = mg = m\frac{GM}{R^2}$$

$$F_g = Ma = M\frac{Gm}{R^2}$$

Free Body Diagram

- The most important step in solving problems involving Newton's Laws is to draw the free body diagram
- Be sure to include only the forces acting on the object of interest
- Include any field forces acting on the object
- Do not assume the normal force equals the weight



Hints for Problem-Solving

- **Read** the problem carefully at least once
- **Draw** a picture of the system, identify the object of primary interest, and indicate forces with arrows
- **Label** each force in the picture in a way that will bring to mind what physical quantity the label stands for (e.g., T for tension)
- **Draw** a free-body diagram of the object of interest, based on the labeled picture. If additional objects are involved, draw separate free-body diagram for them
- **Choose a convenient coordinate system** for each object
- **Apply Newton's second law.** The x- and y-components of Newton second law should be taken from the vector equation and written individually. This often results in two equations and two unknowns
- **Solve** for the desired unknown quantity, and substitute the numbers

$$F_{net,x} = ma_x \qquad F_{net,y} = ma_y$$

Objects in Equilibrium

- Objects that are either at rest or moving with constant velocity are said to be in **equilibrium**
- Acceleration of an object can be modeled as zero:
- Mathematically, the net force acting on the object is zero $\vec{a} = 0$
- Equivalent to the set of component equations given by

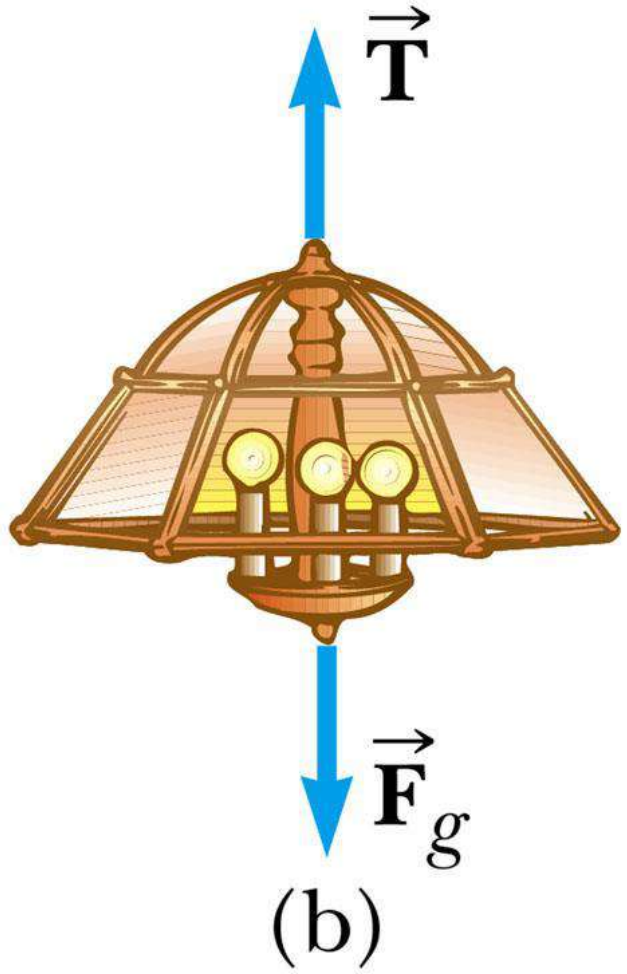
$$\sum \vec{F} = 0$$

$$\sum F_x = 0 \qquad \sum F_y = 0$$

Equilibrium, Example 1

- A lamp is suspended from a chain of negligible mass
- The forces acting on the lamp are
  - the downward force of gravity
  - the upward tension in the chain
- Applying equilibrium gives

$$\sum F_y = 0 \rightarrow T - F_g = 0 \rightarrow T = F_g$$



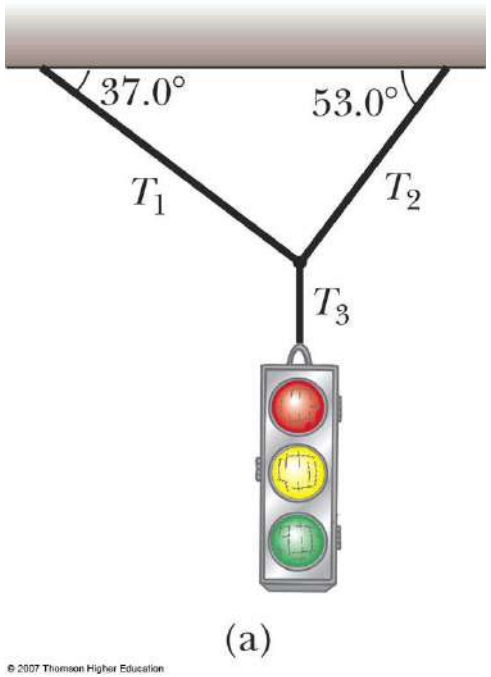
© 2007 Thomson Higher Education

Equilibrium, Example 2

- A traffic light weighing 100 N hangs from a vertical cable tied to two other cables that are fastened to a support. The upper cables make angles of 37° and 53° with the horizontal. Find the tension in each of the three cables.

- Conceptualize the traffic light
  - Assume cables don't break
  - Nothing is moving
- Categorize as an equilibrium problem
  - No movement, so acceleration is zero
  - Model as an object in equilibrium

$$\sum F_x = 0 \qquad \sum F_y = 0$$



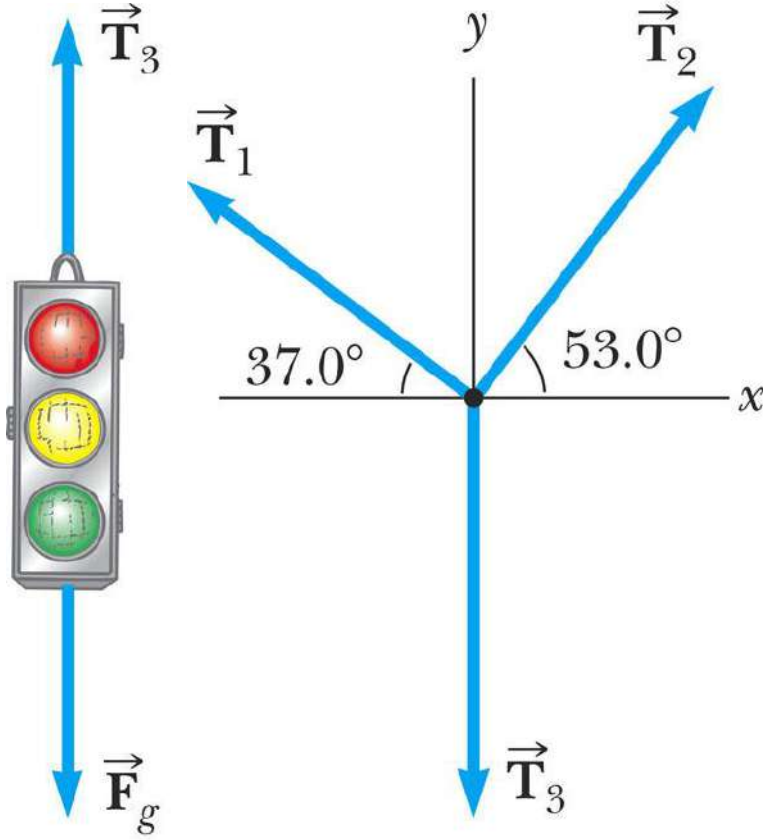
Equilibrium, Example 2

- Need 2 free-body diagrams
  - Apply equilibrium equation to light

$$\sum F_y = 0 \rightarrow T_3 - F_g = 0$$
$$T_3 = F_g = 100N$$

  - Apply equilibrium equations to knot

$$\sum F_x = T_{1x} + T_{2x} = -T_1 \cos 37^\circ + T_2 \cos 53^\circ = 0$$
$$\sum F_y = T_{1y} + T_{2y} + T_{3y}$$
$$= T_1 \sin 37^\circ + T_2 \sin 53^\circ - 100N = 0$$
$$T_2 = T_1\left(\frac{\cos 37^\circ}{\cos 53^\circ}\right) = 1.33T_1$$
$$T_1 = 60N \qquad T_2 = 1.33T_1 = 80N$$



© 2007 Thomson Higher Edu © 2007 Thomson Higher Education

Accelerating Objects

- If an object that can be modeled as a particle experiences an acceleration, there must be a nonzero net force acting on it
- Draw a free-body diagram
- Apply Newton's Second Law in component form

$$\sum \vec{F} = m\vec{a}$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y$$

Accelerating Objects, Example 1

- A man weighs himself with a scale in an elevator. While the elevator is at rest, he measures a weight of 800 N.
  - What weight does the scale read if the elevator accelerates upward at 2.0 m/s^2? a = 2.0 m/s^2
  - What weight does the scale read if the elevator accelerates downward at 2.0 m/s^2? a = - 2.0 m/s^2

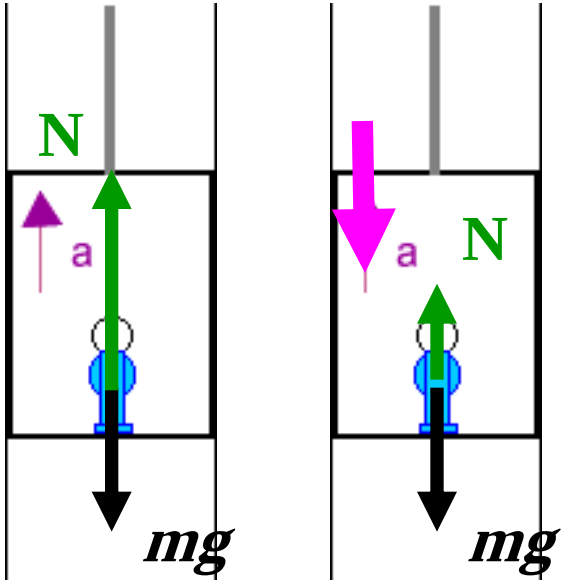
- Upward: $\sum F_y = N - mg = ma$

$N = mg + ma = m(g + a)$ $N = 80(2.0 + 9.8) = 624\,\text{N}$

$m = \frac{w}{g} = \frac{800\,\text{N}}{9.8\,\text{m/s}^2} = 80\,\text{N}$ $N > mg$

- Downward: $N = 80(-2.0 + 9.8) = 624\,\text{N}$

$N < mg$



$9.8) = 1560\,N$

Applications of Newton's Laws

By

Dr. Ammar A. AlRawi

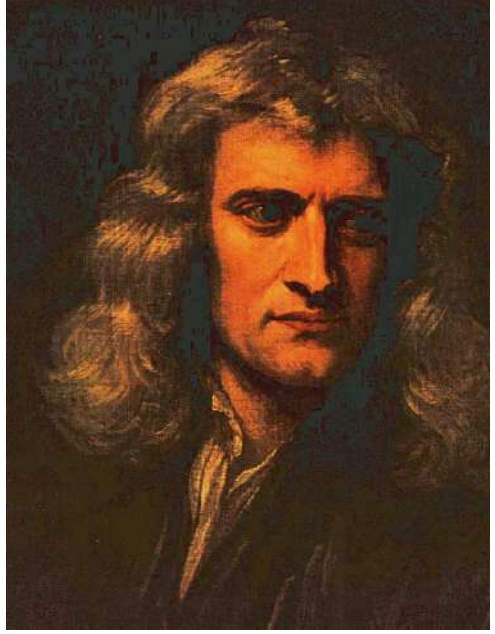
Mathematics and
Computer Applications Department
Collage of Science
2023-2024

Applications of Newton's Laws

- Newton's first law
- Newton's second law
- Newton's third law
- Frictional forces
- Applications of Newton's laws
- Circular Motion



Isaac Newton's work represents one of the greatest contributions to science ever made by an individual.



Force is a vector

Unit of force in S.I.:

$$1N = 1\frac{kg \bullet m}{s^2}$$

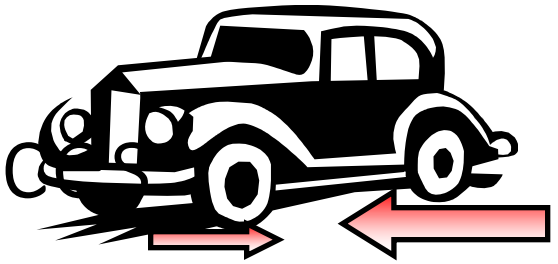
Newton's Laws

I. If no net force acts on a body, then the body's velocity cannot change.

II. The net force on a body is equal to the product of the body's mass and acceleration.

III. When two bodies interact, the force on the bodies from each other are always equal in magnitude and opposite in direction.

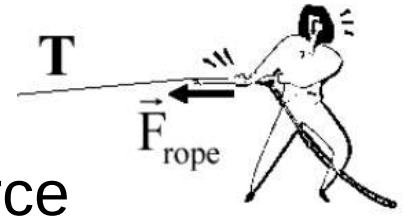
Force



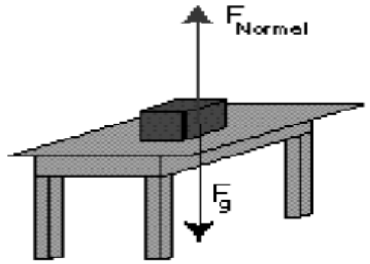
- The measure of interaction between **two objects**
- Vector quantity: has ***magnitude and direction***
- May be a **contact force** or a **field force**

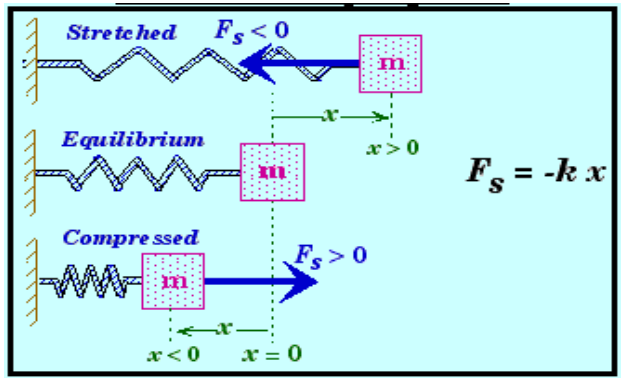


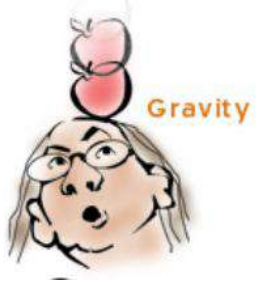




- Particular forces:
  - Gravitational Force
  - Friction Force
  - Tension Force
  - Normal Force
  - Spring Force





Gra

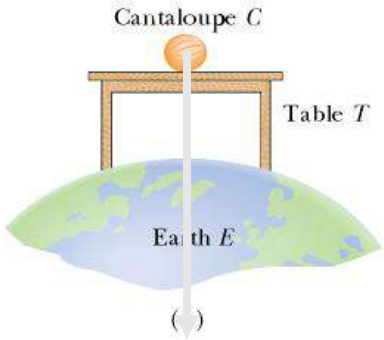
nal Force: ***mg***

- Gravitational force is a vector
- The magnitude of the gravitational force acting on an object of mass m near the Earth's surface is called the weight w of the object

$$F_g = G\frac{mM}{R^2}$$

$$w = mg$$

- Direction: vertically downward



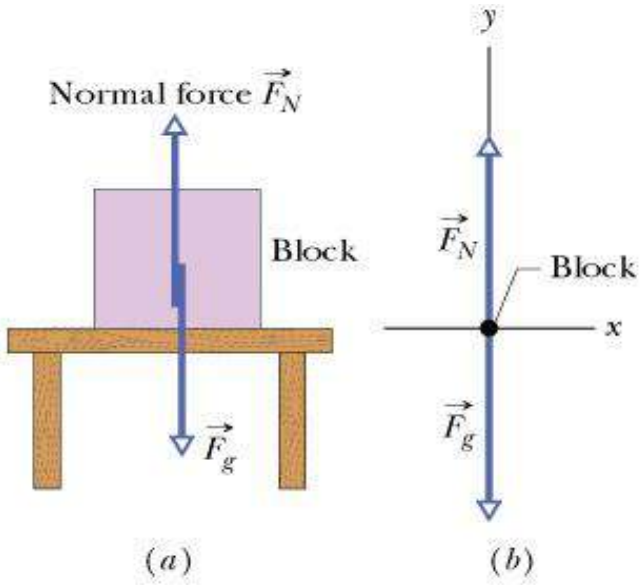
m: Mass

g = 9.8 m/s²



Normal Force: $\boldsymbol{N}$

- Force from a solid surface which keeps object from falling through
- Direction: always perpendicular to the surface
- Magnitude: not necessary to be $\boldsymbol{mg}$



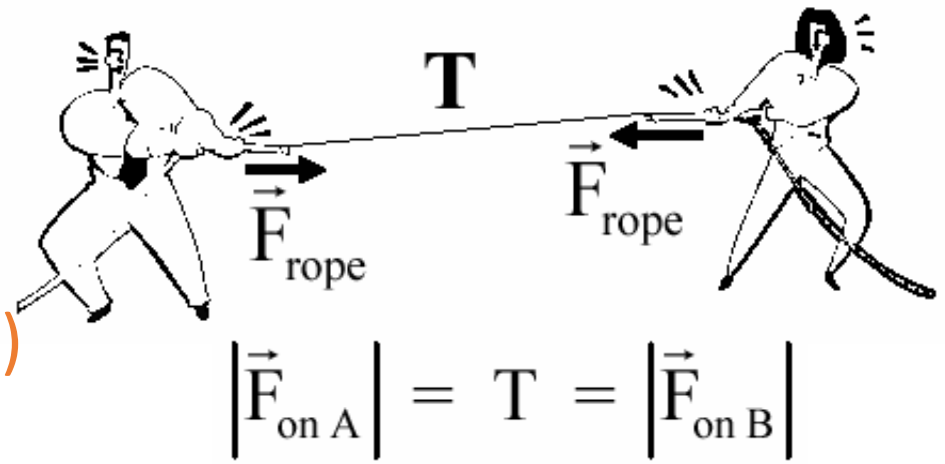
$$N - F_g = ma_y$$

$$N - mg = ma_y$$

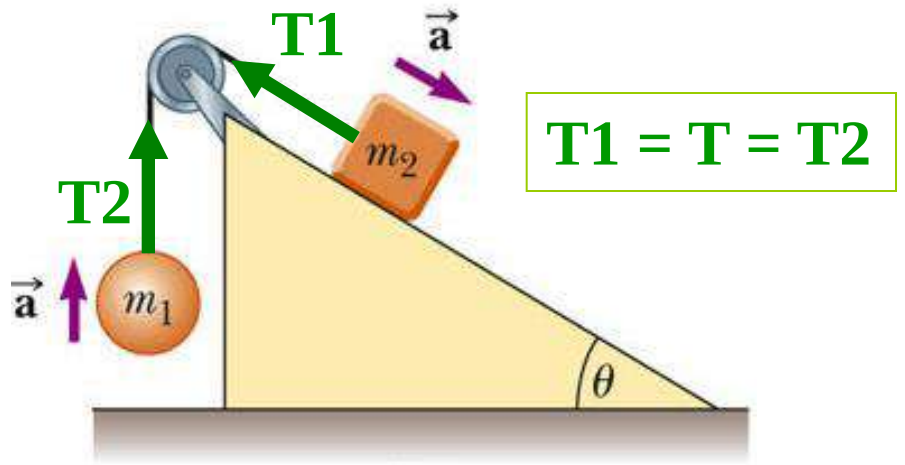
$$N = mg$$

Tension Force: $\boldsymbol{T}$

- A taut rope exerts forces on whatever holds its ends
- Direction: always along the cord (rope, cable, string ……) and away from the object
- Magnitude: depend on situation



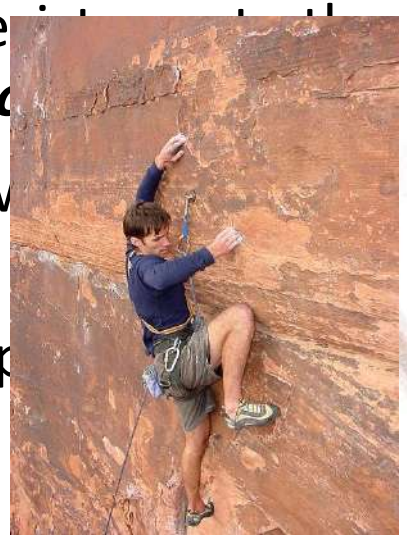
$$\left|\vec{F}_{on\ A}\right| = T = \left|\vec{F}_{on\ B}\right|$$



T1 = T = T2

Forces of Friction: f

- When an object is in motion on a surface or through a viscous medium, there will be a re
  This resistance is called the ***force***
- This is due to the interactions betv
  environment
- We will be concerned with two typ
  - Force of static friction: f_s
  - Force of kinetic friction: f_k





- Direction: opposite the direction of the intended motion
  - If moving: in direction opposite the velocity
  - If stationary, in direction of the vector sum of other forces

Forces of Friction: *Magnitude*

- Magnitude: Friction is proportional to the normal force
  - Static friction: $F_f = F \leq \mu_s N$
  - Kinetic friction: $F_f = \mu_k N$
  - μ is the **coefficient of friction**
- The coefficients of friction are nearly independent of the area of contact (why?)

TABLE 5.1

Coefficients of Friction

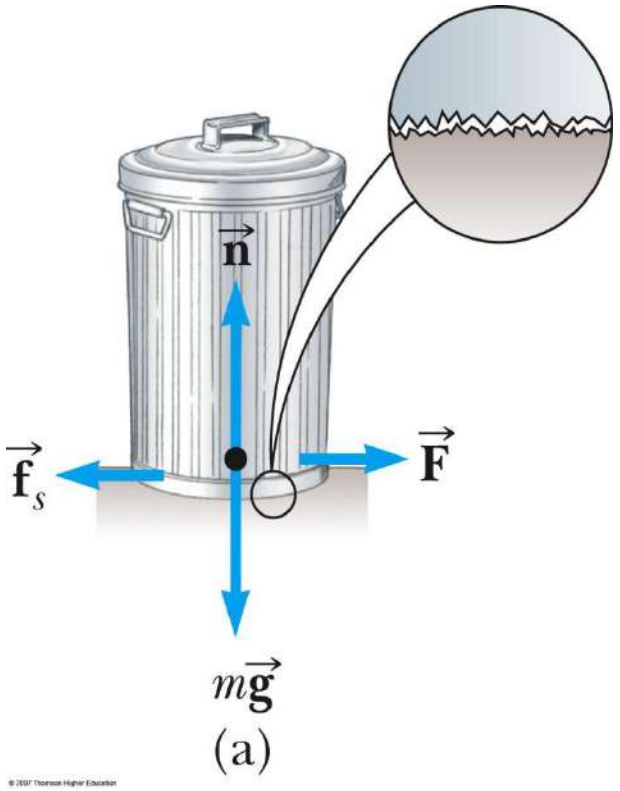
	μ_s	μ_k
Rubber on concrete	1.0	0.8
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Glass on glass	0.94	0.4
Copper on steel	0.53	0.36
Wood on wood	0.25–0.5	0.2
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Teflon on Teflon	0.04	0.04
Ice on ice	0.1	0.03
Synovial joints in humans	0.01	0.003

Note: All values are approximate. In some cases, the coefficient of friction can exceed 1.0.

© 2007 Thomson Higher Education

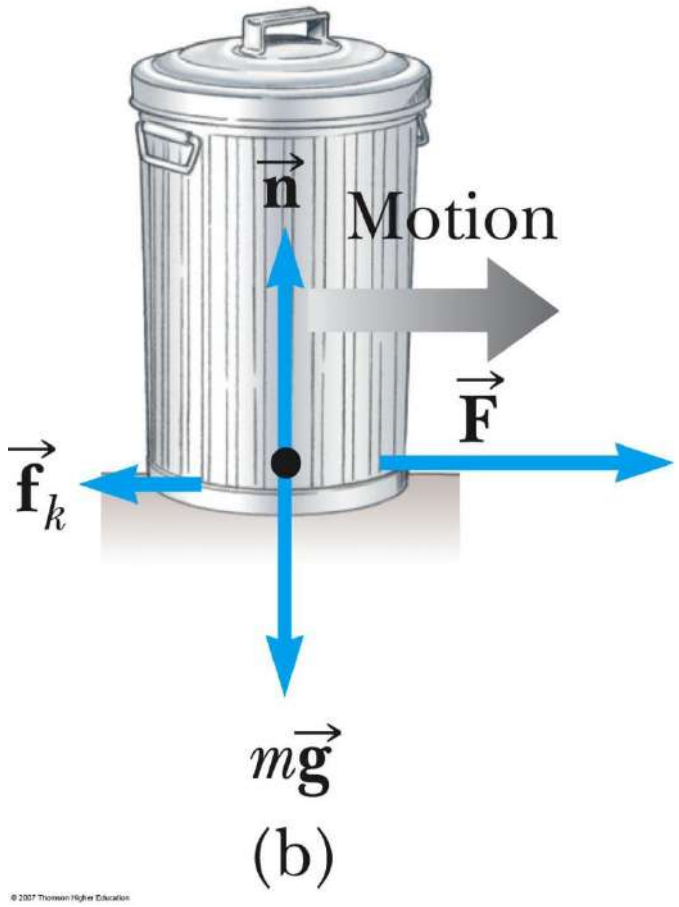
Static Friction

- Static friction acts to keep the object from moving
- If $\vec{F}$ increases, so does $\vec{f}_s$
- If $\vec{F}$ decreases, so does $\vec{f}_s$
- $f_s \leq \mu_s N$
  - Remember, the equality holds when the surfaces are on the verge of slipping



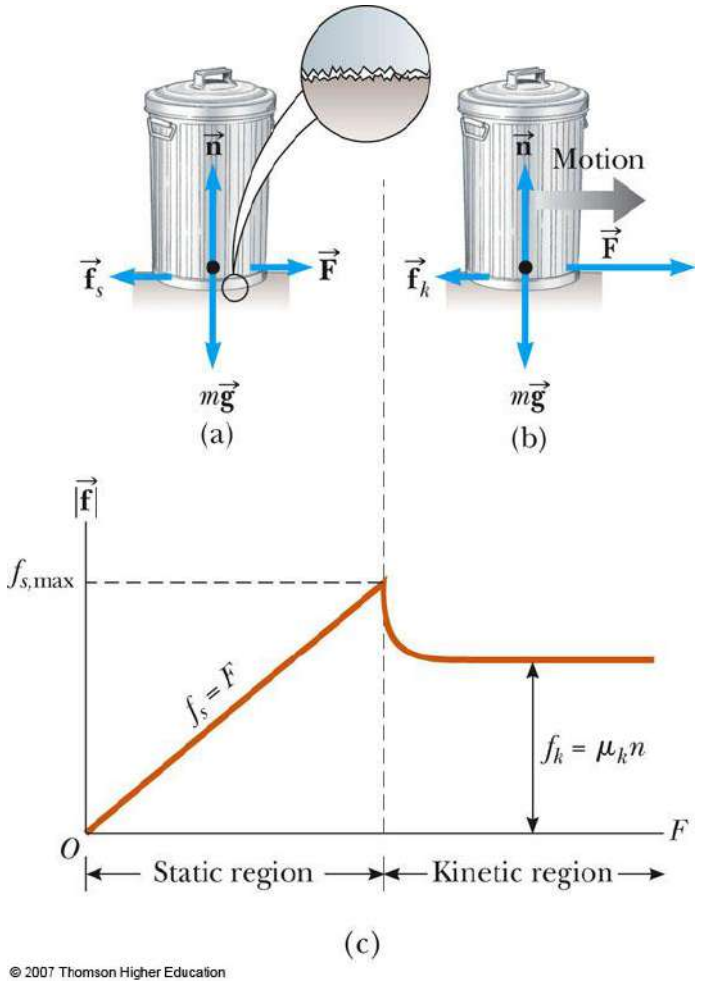
Kinetic Friction

- The force of kinetic friction acts when the object is in motion
- Although μ_k can vary with speed, we shall neglect any such variations
- $f_k = \mu_k N$



Explore Forces of Friction

- Vary the applied force
- Note the value of the frictional force
  - Compare the values
- Note what happens when the can starts to move



© 2007 Thomson Higher Education

Hints for Problem-Solving

- **Read** the problem carefully at least once
- **Draw** a picture of the system, identify the object of primary interest, and indicate forces with arrows
- **Label** each force in the picture in a way that will bring to mind what physical quantity the label stands for (e.g., T for tension)
- **Draw** a free-body diagram of the object of interest, based on the labeled picture. If additional objects are involved, draw separate free-body diagram for them
- **Choose a convenient coordinate system** for each object
- **Apply Newton's second law.** The x- and y-components of Newton second law should be taken from the vector equation and written individually. This often results in two equations and two unknowns
- **Solve** for the desired unknown quantity, and substitute the numbers

$$F_{net,x} = ma_x \qquad F_{net,y} = ma_y$$

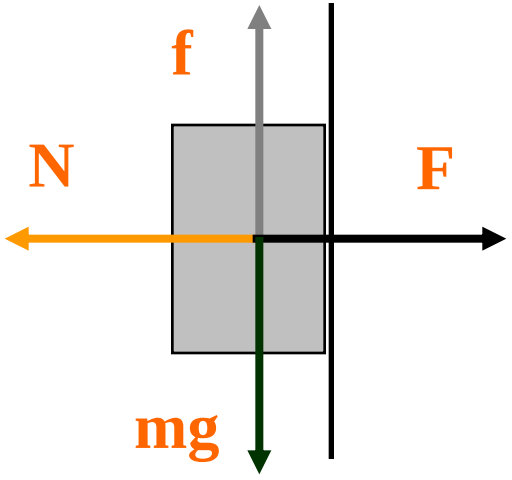
Objects in Equilibrium

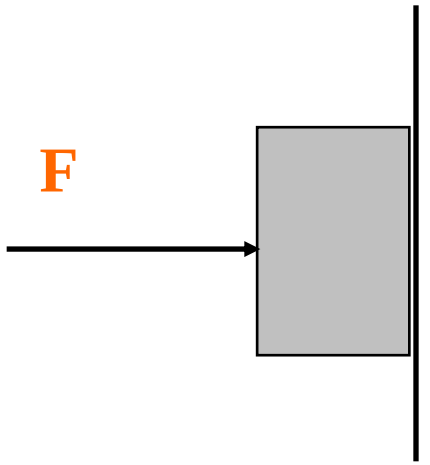
- Objects that are either at rest or moving with constant velocity are said to be in **equilibrium**
- Acceleration of an object can be modeled as zero:
- Mathematically, the net force acting on the object is zero $\vec{a} = 0$
- Equivalent to the set of component equations given by $\sum \vec{F} = 0$

$$\sum F_x = 0 \qquad \sum F_y = 0$$

Equilibrium, Example 1

- What is the smallest value of the force F such that the 2.0-kg block will not slide down the wall? The coefficient of static friction between the block and the wall is 0.2. ?





Accelerating Objects

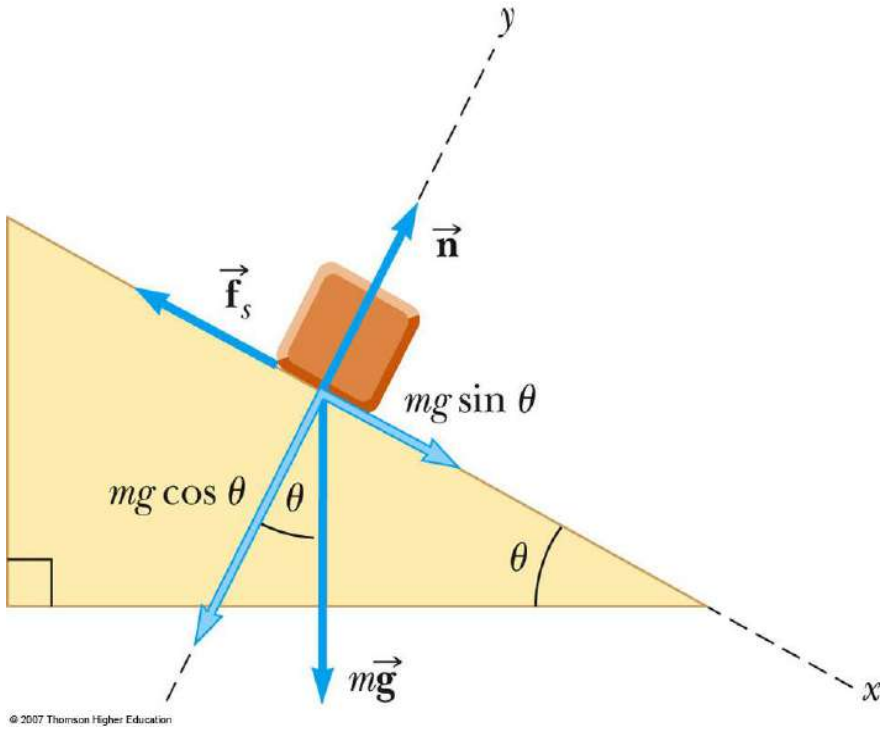
- If an object that can be modeled as a particle experiences an acceleration, there must be a nonzero net force acting on it
- Draw a free-body diagram
- Apply Newton's Second Law in component form

$$\sum \vec{F} = m\vec{a}$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y$$

Inclined Plane

- Suppose a block with a mass of 2.50 kg is resting on a ramp. If the coefficient of static friction between the block and ramp is 0.350, what maximum angle can the ramp make with the horizontal before the block starts to slip down?



Inclined Plane

- Newton 2nd law:

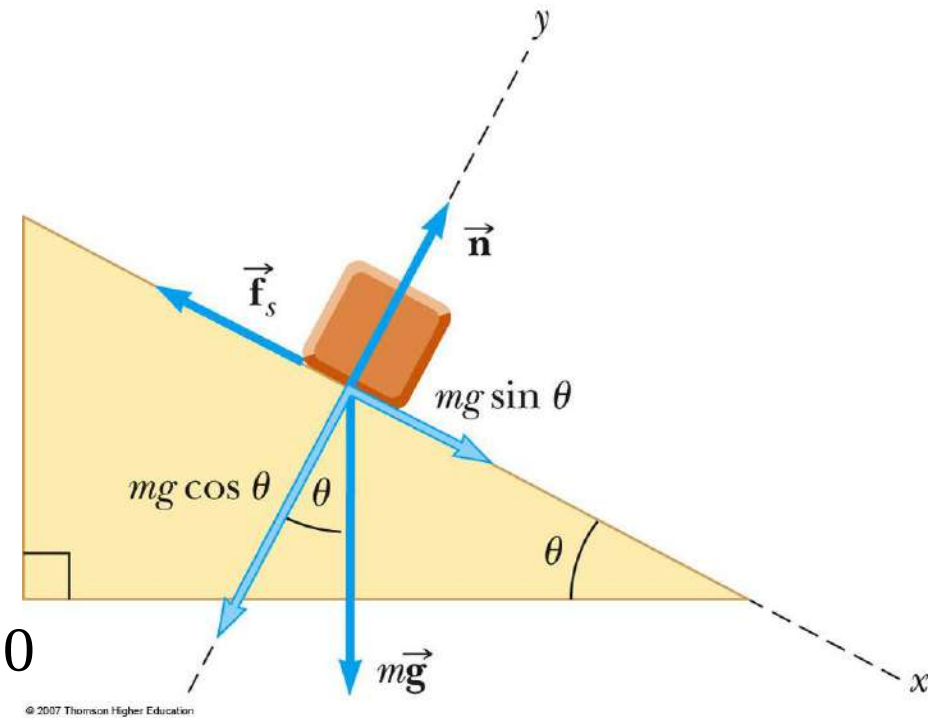
$$\sum F_x = mg\sin\theta - \mu_s N = 0$$
$$\sum F_y = N - mg\cos\theta = 0$$

- Then

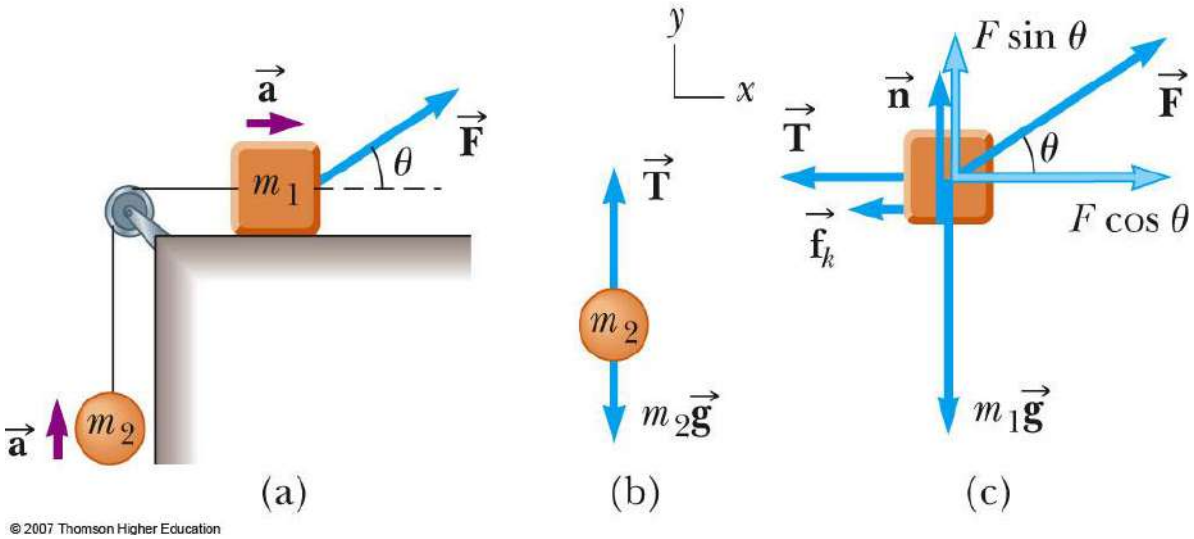
$$N = mg\cos\theta$$
$$\sum F_y = mg\sin\theta - \mu_s mg\cos\theta = 0$$

- So

$$\tan\theta = \mu_s = 0.350$$
$$\theta = \tan^{-1}(0.350) = 19.3^\circ$$



Multiple Objects



© 2007 Thomson Higher Education

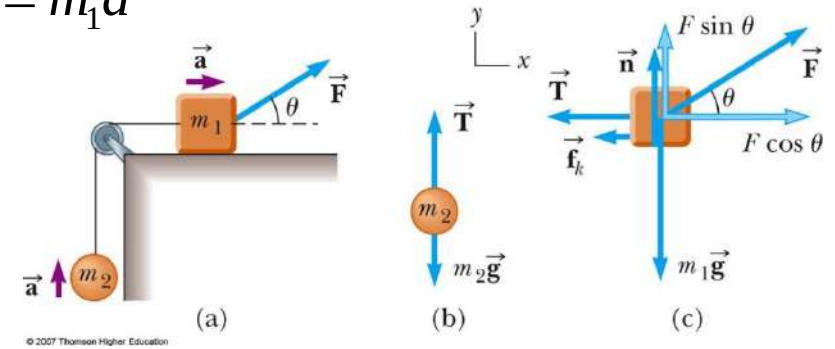
- A block of mass m1 on a rough, horizontal surface is connected to a ball of mass m2 by a lightweight cord over a lightweight, frictionless pulley as shown in figure. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown and the block slides to the right. The coefficient of kinetic friction between the block and surface is μ_k. Find the magnitude of acceleration of the two objects.

Multiple Objects

- m_1: $\sum F_x = F\cos\theta - f_k - T = m_1 a_x = m_1 a$

  $\sum F_y = N + F\sin\theta - m_1 g = 0$

- m_2: $\sum F_y = T - m_2 g = m_2 a_y = m_2 a$



$$T = m_2(a+g)$$

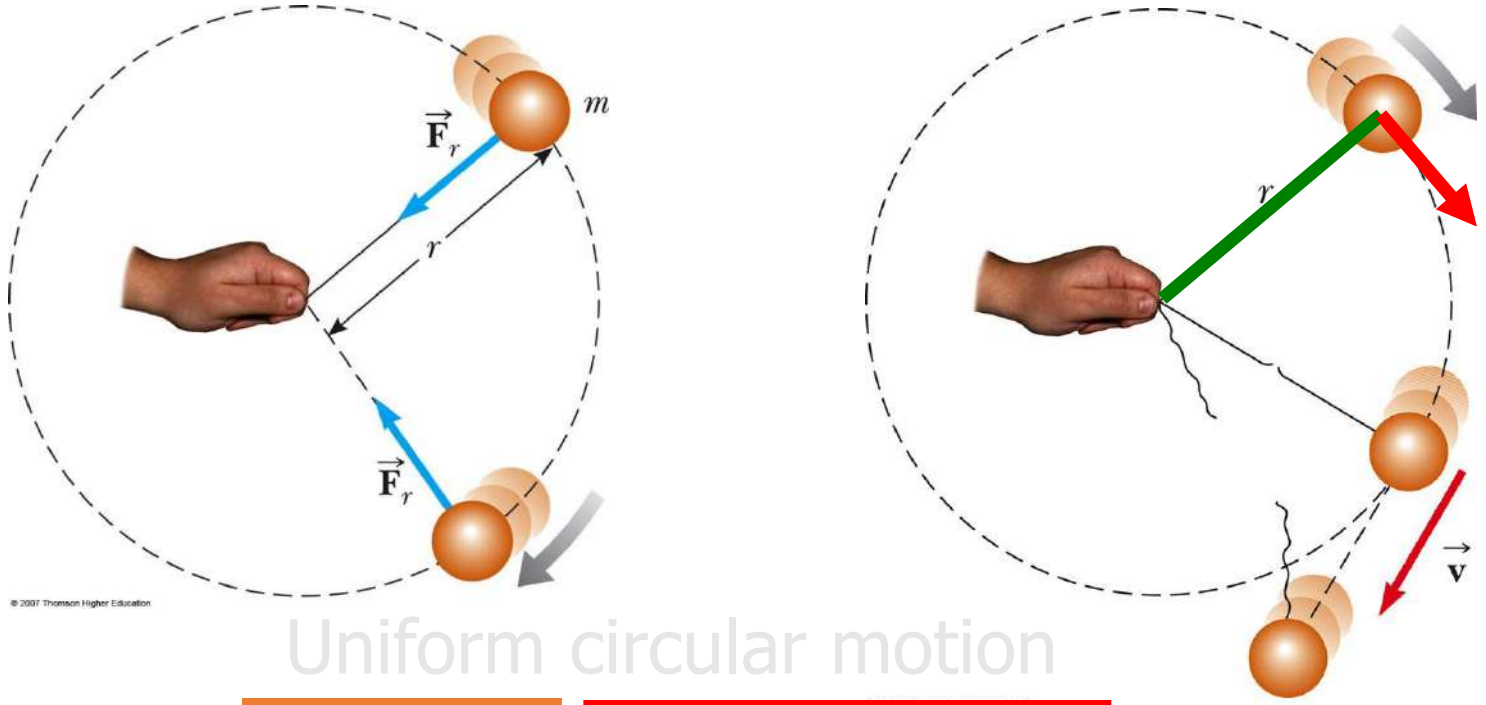
$$N = m_1 g - F\sin\theta$$

$$f_k = \mu_k N = \mu_k(m_1 g - F\sin\theta)$$

$$F\cos\theta - \mu_k(m_1 g - F\sin\theta) - m_2(a+g) = m_1 a$$

$$a = \frac{F(\cos\theta + \mu_k \sin\theta) - (m_2 + \mu_k m_1)g}{m_1 + m_2}$$

Uniform Circular Motion: Definition



© 2007 Thomson Higher Education

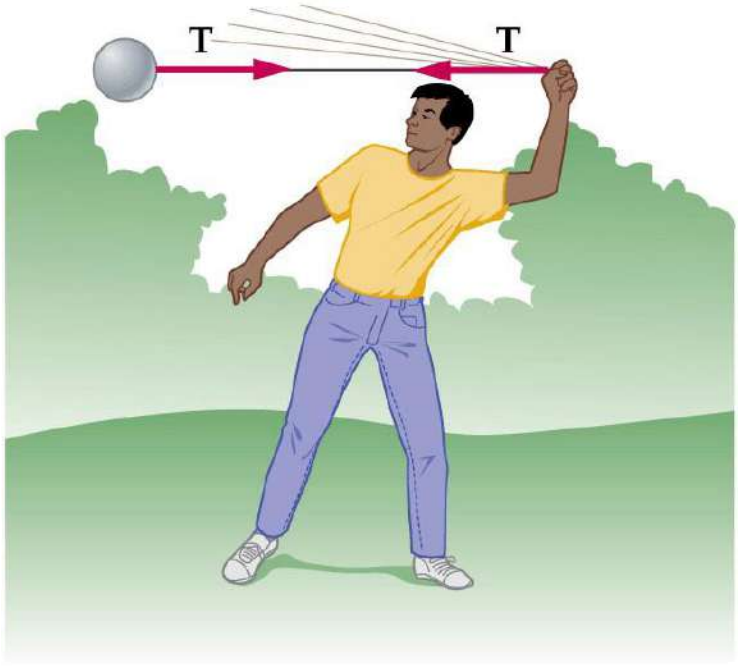
Uniform circular motion

Constant speed, or,
constant magnitude of velocity

Motion along a circle:
Changing direction of velocity

Uniform Circular Motion: Observations

- Object moving along a curved path with **constant speed**
  - Magnitude of velocity: same
  - Direction of velocity: changing
  - Velocity $\vec{v}$: changing
  - Acceleration is NOT zero!
  - **Net force acting on an object is NOT zero**
  - "**Centripetal force**"



$$\vec{F}_{net} = m\vec{a}$$

Uniform Circular Motion

- Magnitude:

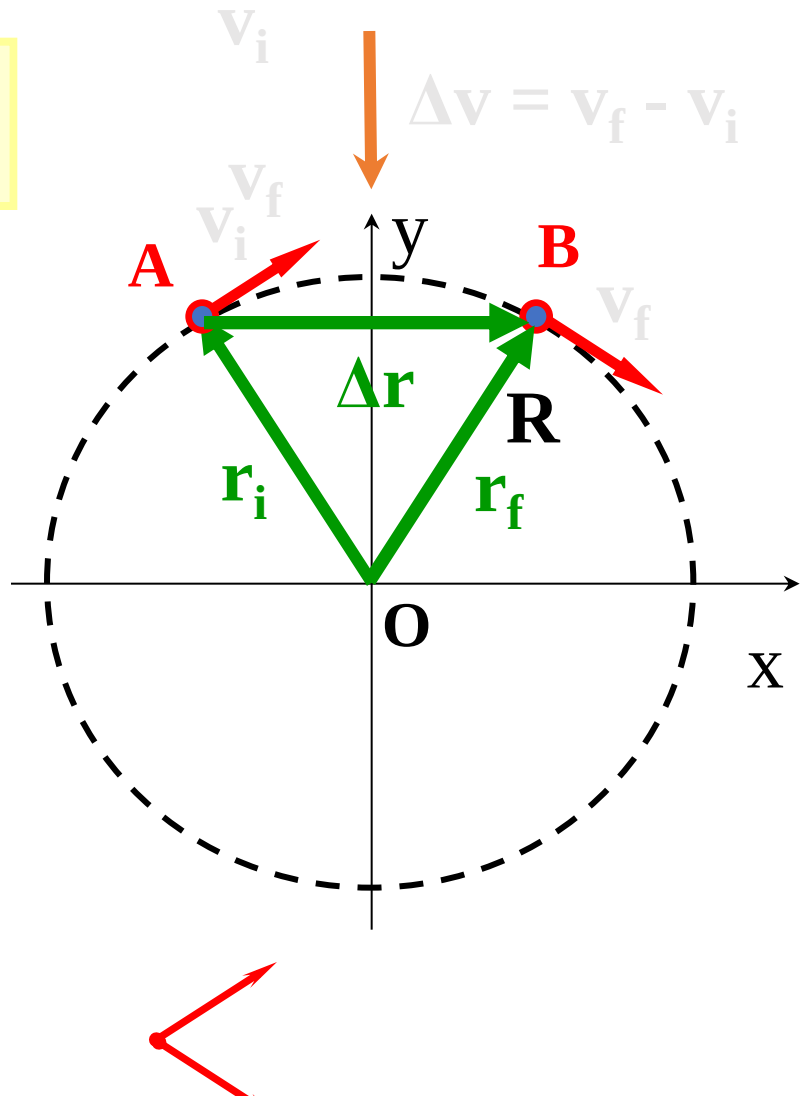
$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i \qquad \vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \quad \text{so,} \quad \Delta v = \frac{v\Delta r}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{\Delta r}{\Delta t}\frac{v}{r} = \frac{v^2}{r}$$

$$a_r = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

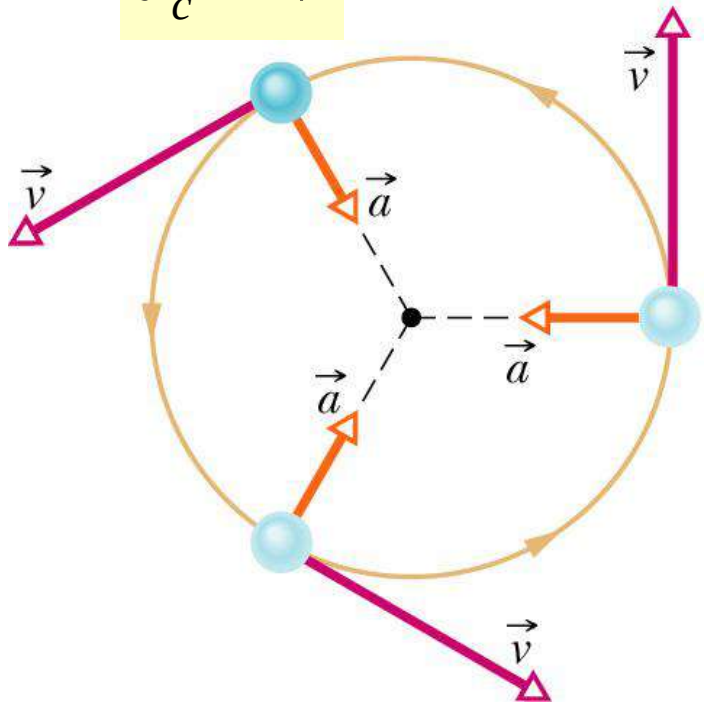
- Direction: **Centripetal**



Uniform Circular Motion

- Velocity:
  - Magnitude: constant v
  - The direction of the velocity is tangent to the circle
- Acceleration:
  - Magnitude: $a_c = \frac{v^2}{r}$
  - directed toward the center of the circle of motion
- Period:
  - time interval required for one complete revolution of the particle $T = \frac{2\pi r}{v}$

$\vec{a}_c \perp \vec{v}$



Centripetal Force

- Acceleration:
  - Magnitude: $a_c = \frac{v^2}{r}$
  - Direction: toward the center of the circle of motion
- Force:
  - Start from Newton's 2nd Law

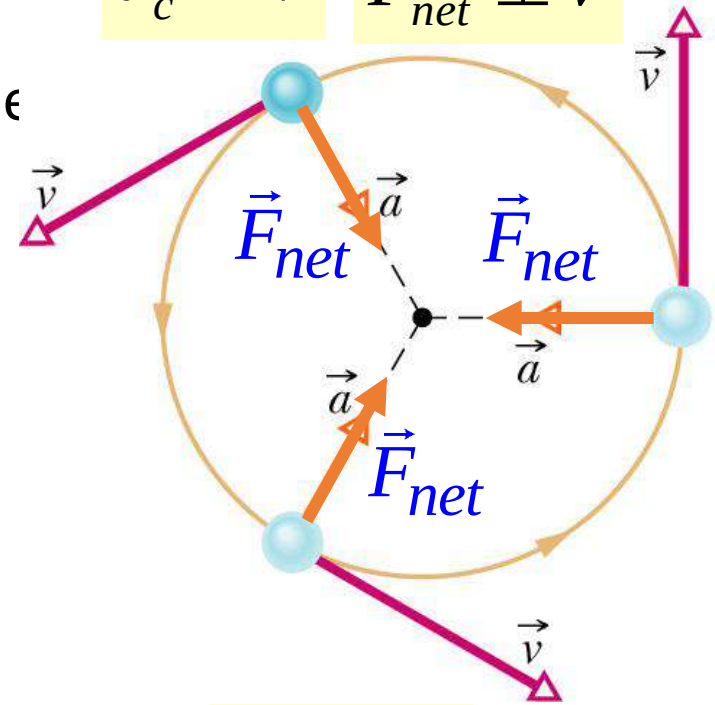
$$\vec{F}_{net} = m\vec{a}$$

  - Magnitude:

$$F_{net} = ma_c = \frac{mv^2}{r}$$

  - Direction: toward the center of the circle of motion

$\vec{a}_c \perp \vec{v}$ $\vec{F}_{net} \perp \vec{v}$



$\vec{a}_c \parallel \vec{F}_{net}$

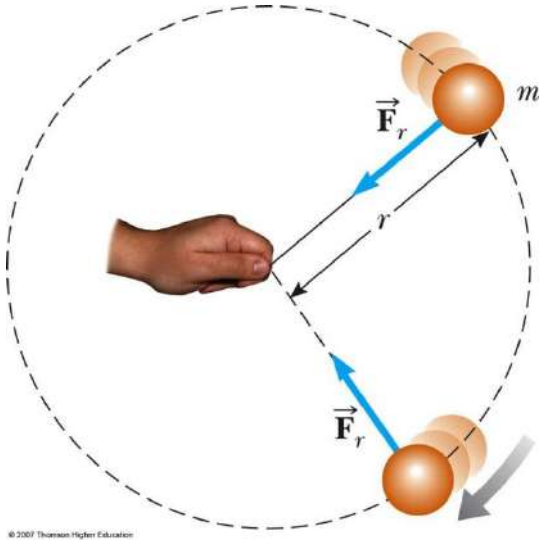
What provides Centripetal Force ?

- Centripetal force is not a new kind of force
- Centripetal force refers to any force that keeps an object following a circular path

$$F_c = ma_c = \frac{mv^2}{r}$$

- Centripetal force is a combination of
  - Gravitational force $m\mathbf{g}$: downward to the ground
  - Normal force $\mathbf{N}$: perpendicular to the surface
  - Tension force $\mathbf{T}$: along the cord and away from object
  - Static friction force: $f_s^{max} = \mu_s N$

What provides Centripetal Force ?

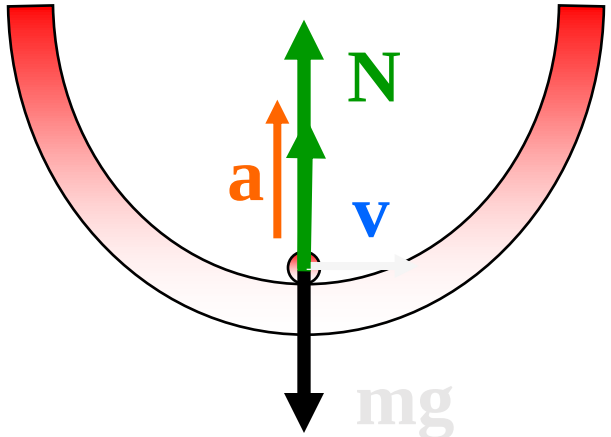


$$F_{net} = T = ma$$

$$T = \frac{mv^2}{r}$$

$$F_{net} = N - mg = ma$$

$$N = mg + m\frac{v^2}{r}$$

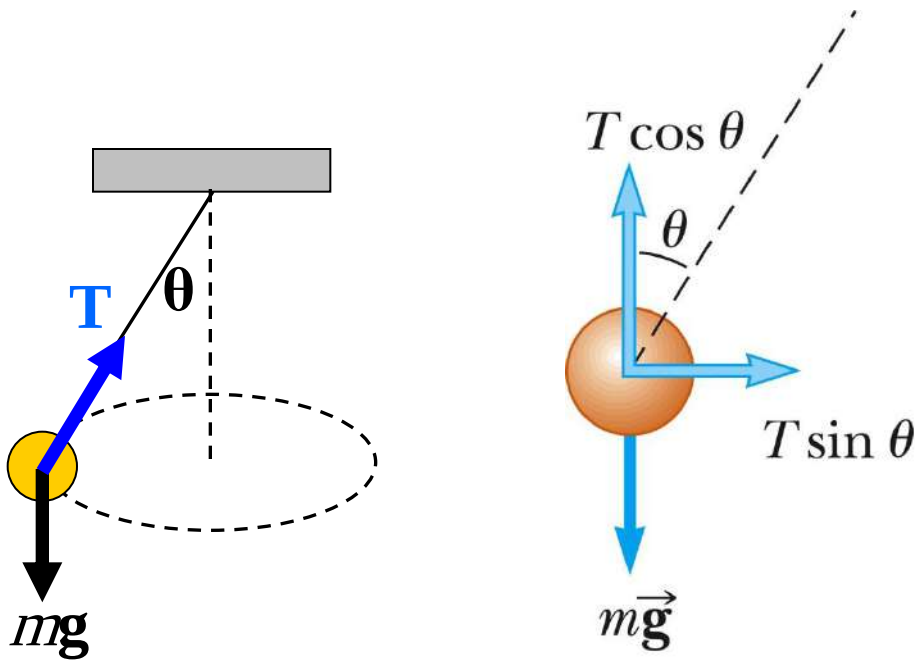


Problem Solving Strategy

- **Draw a free body diagram**, showing and labeling all the forces acting on the object(s)
- **Choose a coordinate system** that has one axis perpendicular to the circular path and the other axis tangent to the circular path
- **Find the net force toward the center** of the circular path (this is the force that causes the centripetal acceleration, F_C)
- **Use Newton's second law**
  - The directions will be radial, normal, and tangential
  - The acceleration in the radial direction will be the centripetal acceleration
- **Solve for the unknown(s)**

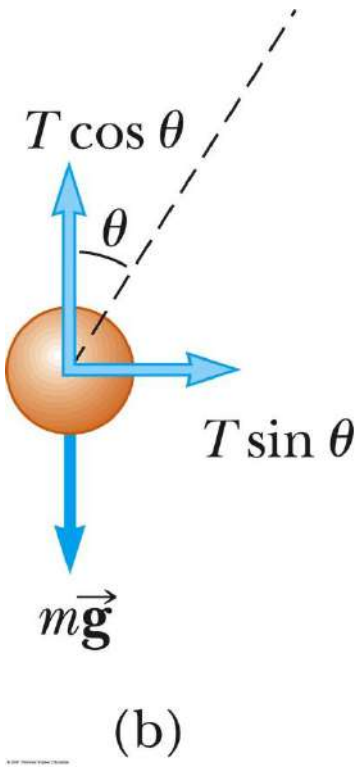
The Conical Pendulum

- A small ball of mass m = 5 kg is suspended from a string of length L = 5 m. The ball revolves with constant speed v in a horizontal circle of radius r = 2 m. Find an expression for v and a.



The Conical Pendulum

Find v and a



$m = 5\,kg \quad L = 5\,m \quad r = 2\,m$

$$\sum F_y = T\cos\theta - mg = 0$$

$$T\cos\theta = mg$$

$$\sum F_x = T\sin\theta = \frac{mv^2}{r}$$

$$\sin\theta = \frac{r}{L} = 0.4$$

$$\tan\theta = \frac{r}{\sqrt{L^2 - r^2}} = 0.44$$

$$T\sin\theta = \frac{mv^2}{r}$$

$$T\cos\theta = mg$$

$$\tan\theta = \frac{v^2}{gr}$$

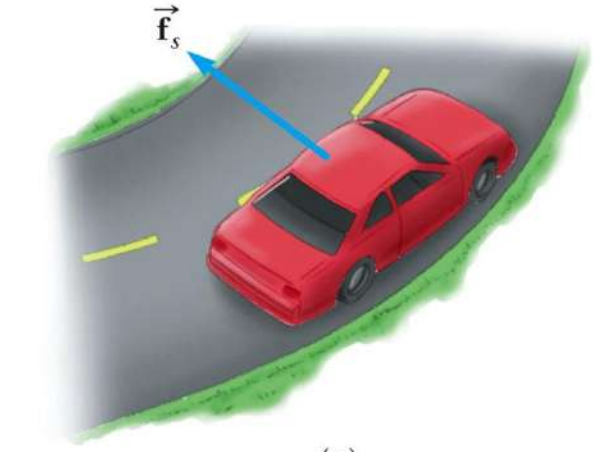
$$v = \sqrt{rg\tan\theta}$$

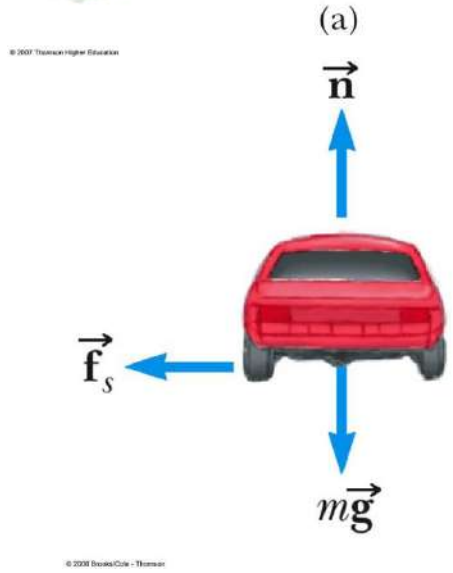
$$v = \sqrt{Lg\sin\theta\tan\theta} = 2.9\ \text{m/s}$$

$$a = \frac{v^2}{r} = g\tan\theta = 4.3\ \text{m/s}^2$$

Level Curves

- A 1500 kg car moving on a flat, horizontal road negotiates a curve as shown. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully.





Level Curves

- The force of static friction directed toward the center of the curve keeps the car moving in a circular path.

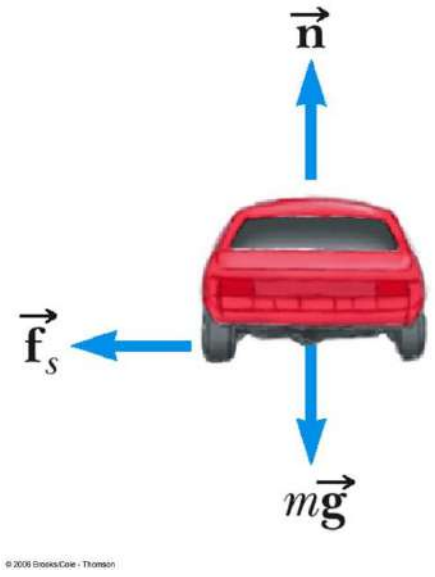
$$f_{s,\max} = \mu_s N = m\frac{v_{\max}^2}{r}$$

$$\sum F_y = N - mg = 0$$

$$N = mg$$

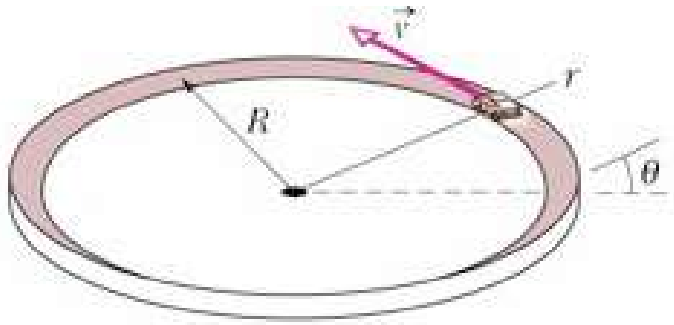
$$v_{\max} = \sqrt{\frac{\mu_s Nr}{m}} = \sqrt{\frac{\mu_s mgr}{m}} = \sqrt{\mu_s gr}$$

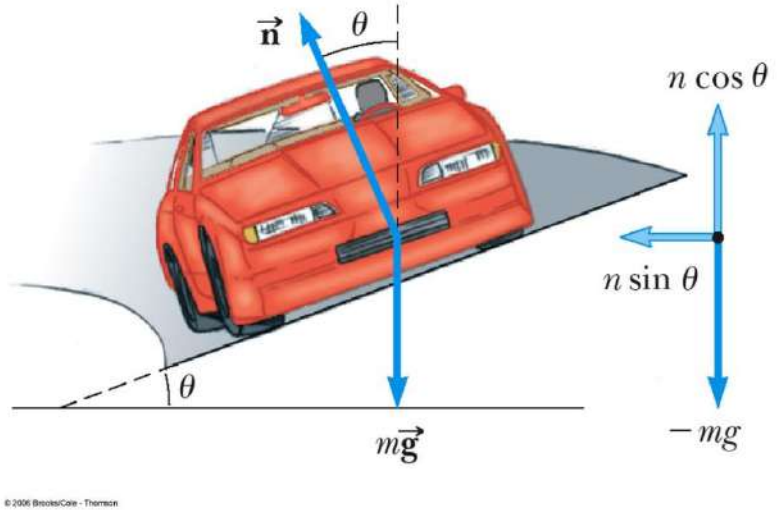
$$= \sqrt{(0.523)(9.8m/s^2)(35.0m)} = 13.4m/s$$



Banked Curves

- A car moving at the designated speed can negotiate the curve. Such a ramp is usually banked, which means that the roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be 13.4 m/s and the radius of the curve is 35.0 m. At what angle should the curve be banked?





© 2006 Brooks/Cole - Thomson

Banked Curves

$v = 13.4\ \text{m/s} \qquad r = 35.0\ \text{m}$

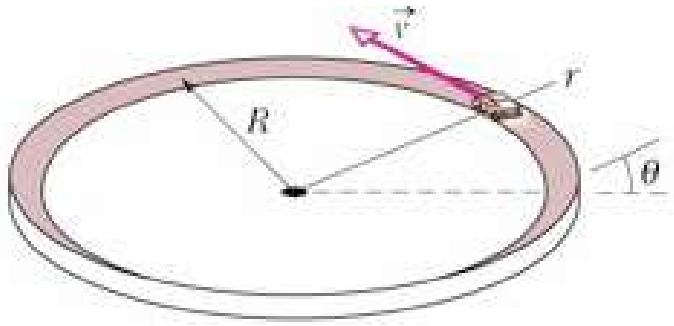
$$\sum F_r = n\sin\theta = ma_c = \frac{mv^2}{r}$$

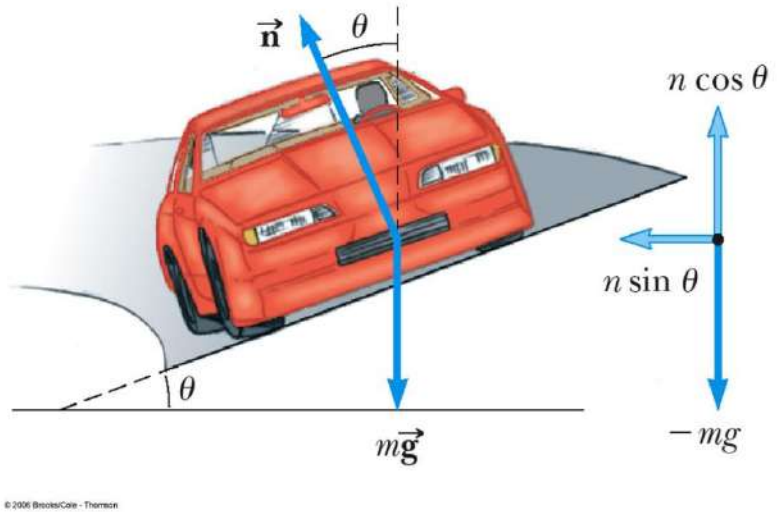
$$\sum F_y = n\cos\theta - mg = 0$$

$$n\cos\theta = mg$$

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{13.4\ \text{m/s}}{(35.0\ \text{m})(9.8\ \text{m/s}^2)}\right) = 27.6^\circ$$





© 2006 Brooks/Cole - Thomson

WORK ENERGY AND POWER

WORK AND THE WORK-ENERGY THEOREM

In this lesson you will…

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.
- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

WORK AND THE WORK-ENERGY THEOREM

- Which of the following is NOT work?
  - Pushing a Stalled Car
  - Pulling a Wagon
  - Climbing stairs
  - Falling Down
  - Carrying a Heavy Backpack Down the Hall

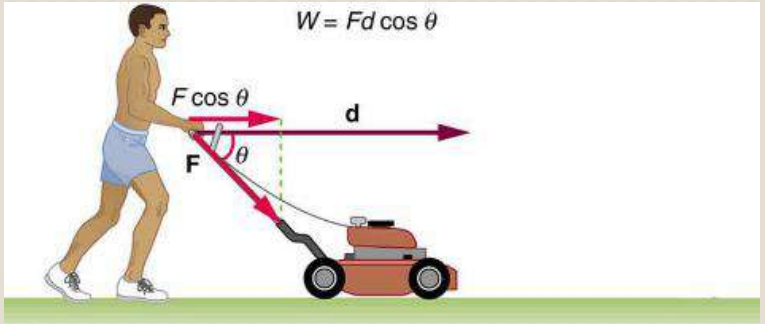
WORK AND THE WORK-ENERGY THEOREM

- Work
  - Depends on the force and the distance the force moves the object.
  - Want the force in the direction of the distance

$$W = \vec{F} \cdot \vec{d}$$

$$W = Fd\cos\theta$$

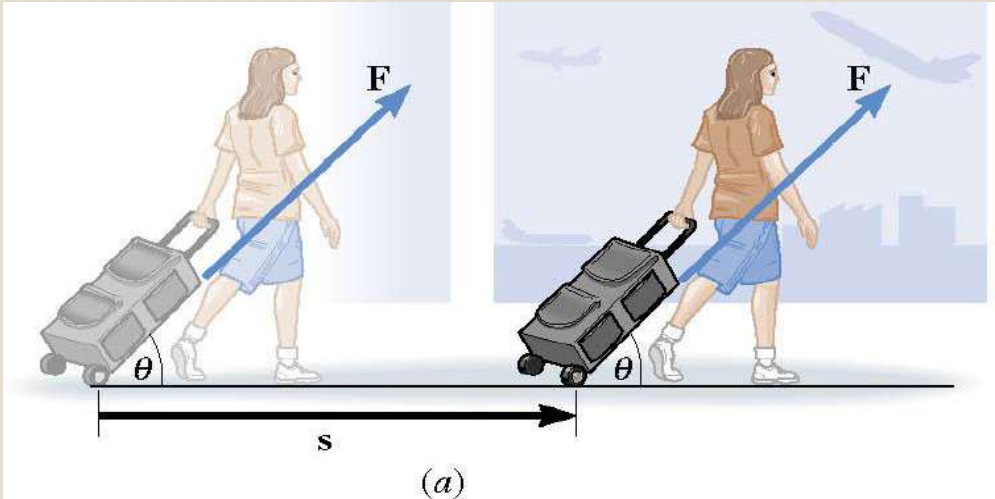
- Unit: $N \cdot m = J$ (Joule) (Scalar)



WORK AND THE WORK-ENERGY THEOREM

- Marcy pulls a backpack on wheels down the 100-m hall. The 60-N force is applied at an angle of 30° above the horizontal. How much work is done by Marcy?

- W = 5200 J



WORK AND THE WORK-ENERGY THEOREM

- Drew is carrying books (200 N) down the 100-m hall. How much work is Drew doing on the books?

- W = 0 J

- The force is vertical displacement is horizontal.



WORK AND THE WORK-ENERGY THEOREM

- You carry some books (200 N) while walking down stairs height 2 m and length 3 m. How much work do you do?

- W = -400 J



WORK AND THE WORK-ENERGY THEOREM

- A suitcase is hanging straight down from your hand as you ride an escalator. Your hand exerts a force on the suitcase, and this force does work. Which one of the following is correct?
- The W is negative when you ride up and positive when you ride down
- The W is positive when you ride up and negative when you ride down
- The W is positive
- The W is negative

WORK AND THE WORK-ENERGY THEOREM

- Do work means → W = Fd
  - F = ma

- So work by a net force gives an object some acceleration

- Acceleration means the velocity changes

WORK AND THE WORK-ENERGY THEOREM

- $F = ma$
- $Fd = mad$
- ${v_f}^2 = {v_0}^2 + 2ad$ → solve for ad
  - $ad = \frac{1}{2}\left(v_f^2 - v_0^2\right)$
- $Fd = m\frac{1}{2}\left(v_f^2 - v_0^2\right)$

- $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

WORK AND THE WORK-ENERGY THEOREM

- Energy is the ability to do work
- Kinetic Energy - Energy due to motion
  - If something in motion hits an object, it will move it some distance

- $KE = \frac{1}{2}mv^2$
  - Scalar
  - Unit is joule (J)

WORK AND THE WORK-ENERGY THEOREM

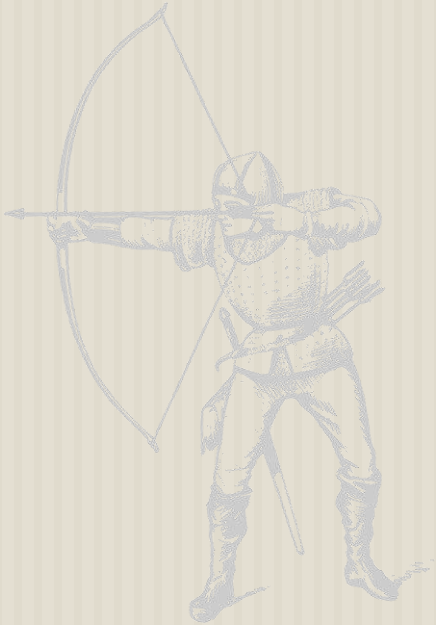
- **Work Energy Theorem**
- **Work of Net external force** = change in kinetic energy

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

$$W = KE_f - KE_0$$

WORK AND THE WORK-ENERGY THEOREM

- A 0.075-kg arrow is fired horizontally. The bowstring exerts a force on the arrow over a distance of 0.90 m. The arrow leaves the bow at 40 m/s. What average force does the bow apply to arrow?



POTENTIAL ENERGY AND CONSERVATIVE FORCES

In this lesson you will…

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass *m* at height *h* on Earth is given by PEg = *mgh*.
- Define conservative force, potential energy, and mechanical energy.
- Explain the potential energy of a spring in terms of its compression when Hooke's law applies.
- Use the work-energy theorem to show how having only conservative forces implies conservation of mechanical energy.

POTENTIAL ENERGY AND CONSERVATIVE FORCES

- Potential energy
  - Energy due to position
- $W = Fd$
- Gravity
- $W_{gravity} = mgh$

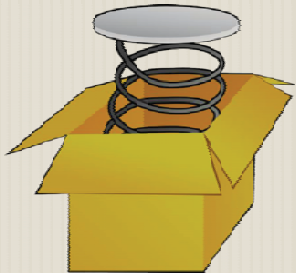
$$PE = mgh$$

- Since the force of gravity is down
  - We only worry about the vertical distance
- Potential Energy is not absolute
  - It is a difference
- The path the object takes doesn't matter, just the vertical distance
- h is measured from any chosen point. Just be consistent

POTENTIAL ENERGY AND CONSERVATIVE FORCES

- Spring Potential Energy

$$PE_S = \frac{1}{2}kx^2$$



POTENTIAL ENERGY AND CONSERVATIVE FORCES

- Conservative Forces
  - A force where the work it does is independent of the path
    - Only thing that matters is starting and stopping point

POTENTIAL ENERGY AND CONSERVATIVE FORCES

- Examples of conservative forces
  - Gravitational Force
  - Elastic Spring Force
  - Electric Force
- Examples of Nonconservative forces
  - Friction
  - Air resistance
  - Tension
  - Normal force
  - Propulsion force of things like rocket engine
- Each of these forces depends on the path

POTENTIAL ENERGY AND CONSERVATIVE FORCES

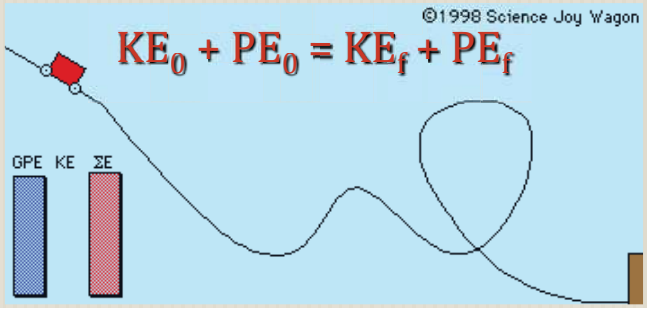
- Potential energy can be converted into Kinetic energy and back
- Think of an object thrown up
- Bottom → 0 PE, high KE
- Top → high PE, 0 KE
- $\Delta KE = -\Delta PE$
- $KE_f - KE_0 = -(PE_f - PE_0)$
- Rearrange
- Conservation of Mechanical Energy

$$KE_f + PE_f = KE_0 + PE_0$$

  - if only conservative forces do net work

POTENTIAL ENERGY AND CONSERVATIVE FORCES

- If there is no work done by nonconservative forces
- Total mechanical energy is constant



POTENTIAL ENERGY AND CONSERVATIVE FORCES

- A toy gun uses a spring to shoot plastic balls ($m = 50\ g$). The spring is compressed by 3.0 cm. Let $k = 2.22 \times 10^5\ N/m$.
- (a) Of course, you have to do some work on the gun to arm it. How much work do you have to do?
- (b) Suppose you fire the gun horizontally. How fast does the ball leave the gun?
- (c) Now suppose you fire the gun straight upward. How high does the ball go?

POTENTIAL ENERGY AND CONSERVATIVE FORCES

- A 1500-kg car is driven off a 50-m cliff during a movie stunt. If it was going 20 m/s as it went off the cliff, how fast is it going as it hits the ground?

POWER

In this lesson you will…

- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

POWER

- Two cars with the same mass do the same amount of work to get to 100 km/h.
- Which car is better
  - Takes 8.0 s
  - Takes 6.2 s
- Sometimes the time taken to do the work is important

03-04 POWER

- Rate that work is done

$$P = \frac{W}{t}$$

- Unit: joule/s = **watt (W)**

POWER

- Since work changes the amount of energy in an object
- Power is the rate that energy is changing

POWER

- A 1000 kg car accelerates from 0 to 100 km/h in 3.2 s on a level road. Find the average power of the car.
- P = 121000 W

