Electric Charge and Electric Field

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Lecture Overview

01 **Introduction to Electric Charge**
Historical discovery and everyday examples

02 **Types of Electric Charge**
Positive, negative, and their interactions

03 **Conservation of Charge**
Fundamental principle and applications

04 **Conductors and Insulators**
Material properties and charge movement

05 **Coulomb's Law**
Mathematical formulation and calculations

06 **Electric Field Concept**
Definition, visualization, and properties

07 **Electric Field Calculations**
Point charges and superposition principle

08 **Summary and Review**
Key takeaways and practice guidance

Section 1

What is Electric Charge?

Historical Discovery

The ancient Greeks discovered as early as **600 B.C.** that after rubbing amber with wool, the amber could attract other objects. The word "electric" comes from the Greek word elektron, meaning amber.

Everyday Examples

- Scuffing your shoes across a nylon carpet
- Passing a comb through dry hair
- Feeling a shock when touching a doorknob

When objects become charged, they can exert forces on other objects without touching them!

Electric Charge Definition

Electric charge is a fundamental property of matter that causes it to experience a force when placed in an electromagnetic field.

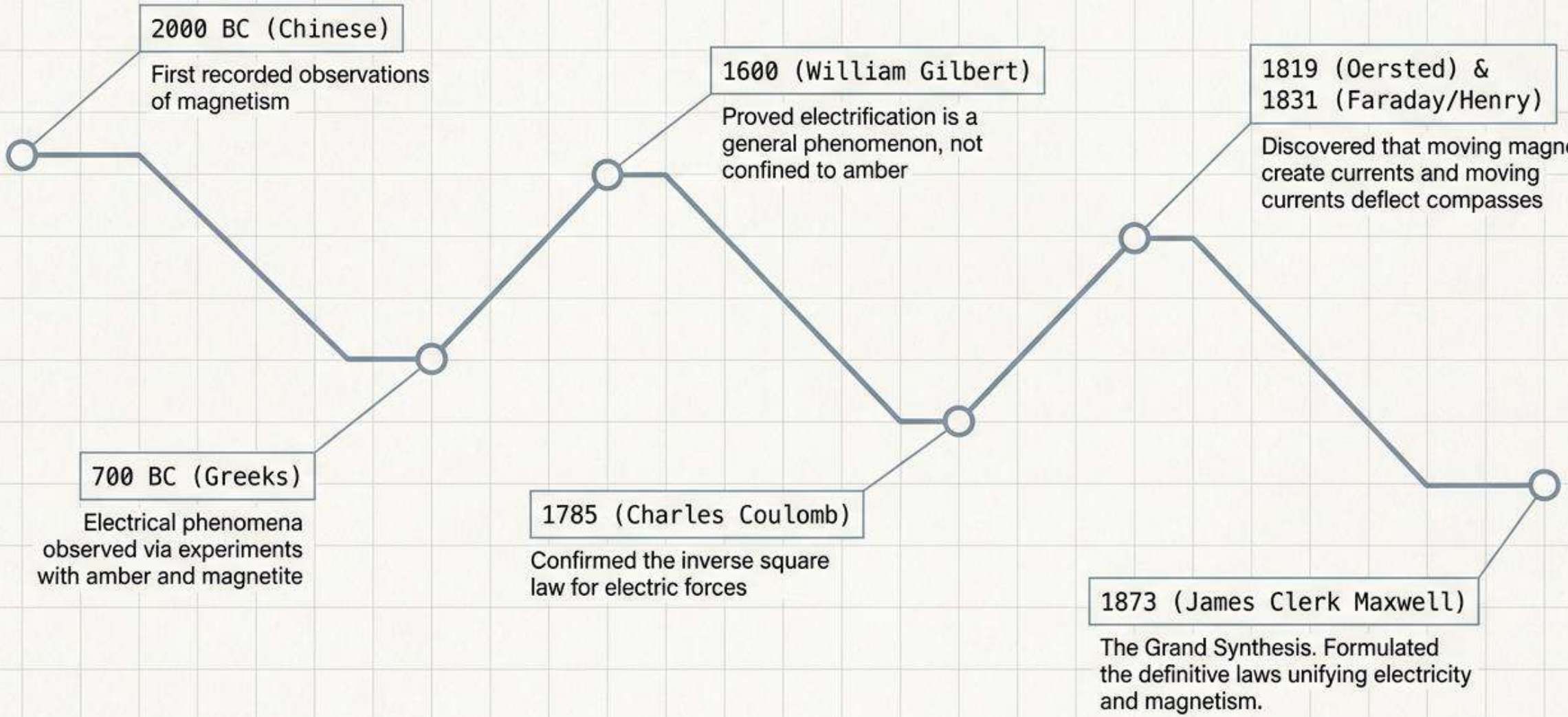


Opposite charges attract each other

Interactive Question

What happens when you rub a plastic rod with fur?

The Long Path to Unification



± Section 2

The Two Types of Charge

+ Positive

- Found on **glass rods** rubbed with silk
- Associated with protons
- Repels other positive charges

− Negative

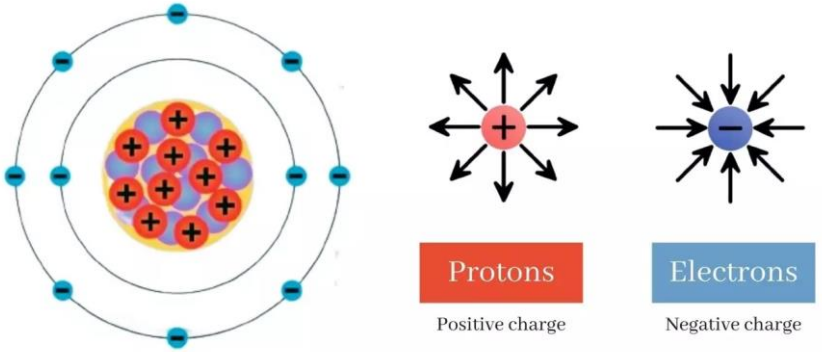
- Found on **plastic rods** rubbed with fur
- Associated with electrons
- Repels other negative charges

The Fundamental Rule

Like charges repel, opposite charges attract

+ +	− −	+ −
Repel ↑↑	Repel ↑↑	Attract →←

Charge Interactions



Visual representation of charge interactions

Experimental Evidence

Plastic + Plastic	Glass + Glass	Plastic + Glass
Repel	Repel	Attract

Test Your Knowledge

Two charged objects repel each other. What can you conclude?

Section 3

Atomic Structure and Charge

Three Fundamental Particles

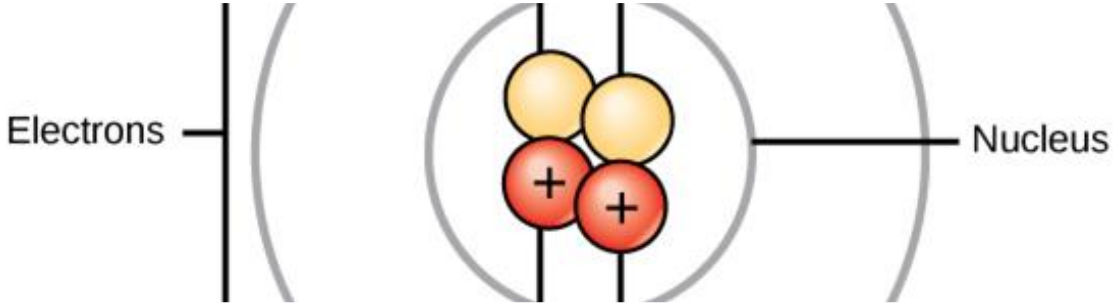
Proton
Positive charge: $+e = +1.602 \times 10^{-19}$ C
Mass: 1.673×10^{-27} kg (in nucleus)

Electron
Negative charge: $-e = -1.602 \times 10^{-19}$ C
Mass: 9.109×10^{-31} kg (orbiting nucleus)

Neutron
No charge (neutral)
Mass: 1.675×10^{-27} kg (in nucleus)

Proton and neutron masses are nearly equal and about 2000 times the electron mass!

Atom Structure



Nucleus: $\sim 10^{-15}$ m (protons + neutrons)
Electrons: Orbit at $\sim 10^{-10}$ m

Types of Ions

Positive Ion
Lost electrons
More protons than electrons

Negative Ion
Gained electrons
More electrons than protons

Quick Check

What makes an atom electrically neutral?

Section 4

Conservation of Charge

Fundamental Principle

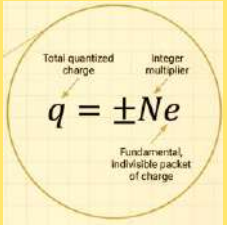
The algebraic sum of all electric charges in any closed system is **constant**.

What This Means

- Charge is **not created or destroyed** in any process
- Charge is only **transferred** from one body to another
- Total charge before = Total charge after

Charge Quantization

Every observable charge is an **integer multiple** of the elementary charge $e = 1.602 \times 10^{-19}$ C. We say charge is **quantized**.



Example: Charging by Friction

Rod	+	Fur	→	Rod −	+	Fur +
Initially: 0		Initially: 0		Gains e^-		Loses e^-

Total Charge: 0 + 0 = (−) + (+) = 0 ✓

The rod becomes negative, fur becomes positive, but total charge remains zero!

Think About It

If a neutral object gains 5 electrons, what is its charge?

Section 5

Conductors and Insulators

Conductors

Definition: Materials that permit the easy movement of electric charge through them.

Mechanism:

One or more outer electrons in each atom become **detached** and can move freely throughout the material—like molecules of gas moving through sand.

Examples:

Copper | Silver | Aluminum | Gold

Insulators

Definition: Materials that do NOT permit the easy movement of electric charge.

Mechanism:

Electrons are **tightly bound** to their atoms. There are no (or very few) free electrons to move through the material.

Examples:

Rubber | Glass | Plastic | Nylon

Key Comparison

Property	Conductors	Insulators
Free electrons	Many	None/Few
Charge movement	Easy	Difficult
Examples	Metals	Rubber, Glass

Everyday Application

Why do you get shocked touching a doorknob?

1. Walking on carpet charges you up (insulator)
2. Charge stays on you (can't flow through carpet)
3. Touching metal doorknob (conductor)
4. Charge rapidly transfers → **SHOCK!**

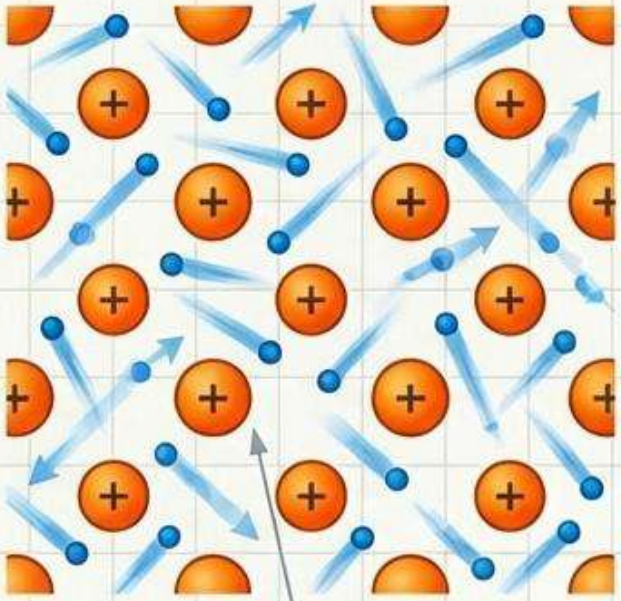
Test Yourself

Why are electrical wires made of copper?

Section 5

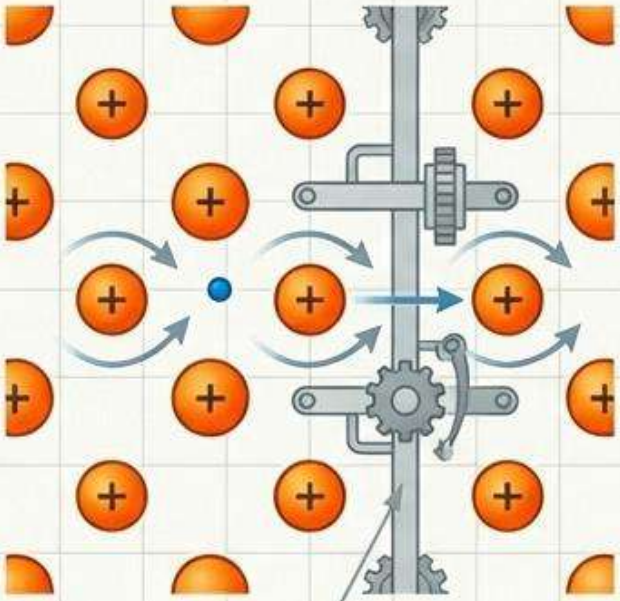
Conductors and Insulators

Conductors (Copper, Aluminum, Silver)



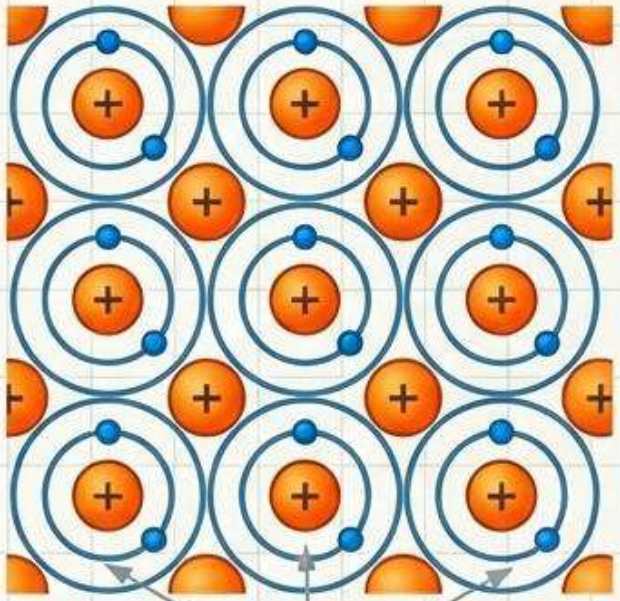
Free electrons are unbound and distribute readily over the entire material surface.

Semiconductors (Silicon, Germanium)



Properties sit precisely between conductors and insulators. Conductivity is altered by adding controlled amounts of specific atoms.

Insulators (Glass, Rubber, Wood)



All electrons are tightly bound. Charge remains locked exactly where it is placed.

Section 6

Charging by Induction

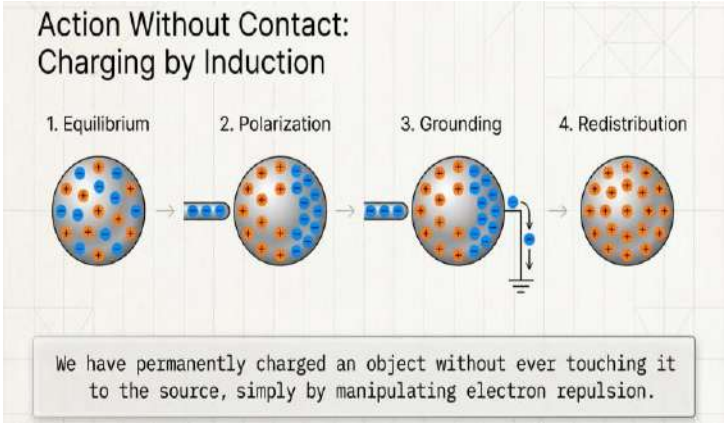
What is Induction?

A method to charge a conductor **without direct contact**. A charged object induces charge separation in a neutral conductor, which can then be permanently charged by grounding.

Step-by-Step Process

1. **Neutral metal ball on insulating stand**
   Equal positive and negative charges
2. **Bring negatively charged rod near (don't touch)**
   Electrons repelled to far side → induced charges
3. **Ground the metal ball (connect to earth)**
   Excess electrons flow to ground
4. **Remove ground connection**
   Ball now has electron deficiency
5. **Remove charged rod**
   Ball now has **NET POSITIVE CHARGE!**

Visual Diagram



Key Insight

The rod **never loses any of its charge** during induction! The ball becomes oppositely charged through redistribution and grounding.

Section 7

Coulomb's Law - The Foundation

Charles Augustin de Coulomb (1736-1806)

French physicist who studied electrostatic forces using a **torsion balance** in 1784. Discovered the fundamental law governing electric forces between charged particles.

Coulomb's Law Statement

The magnitude of the electric force between two point charges is **directly proportional** to the product of the charges and **inversely proportional** to the square of the distance between them.

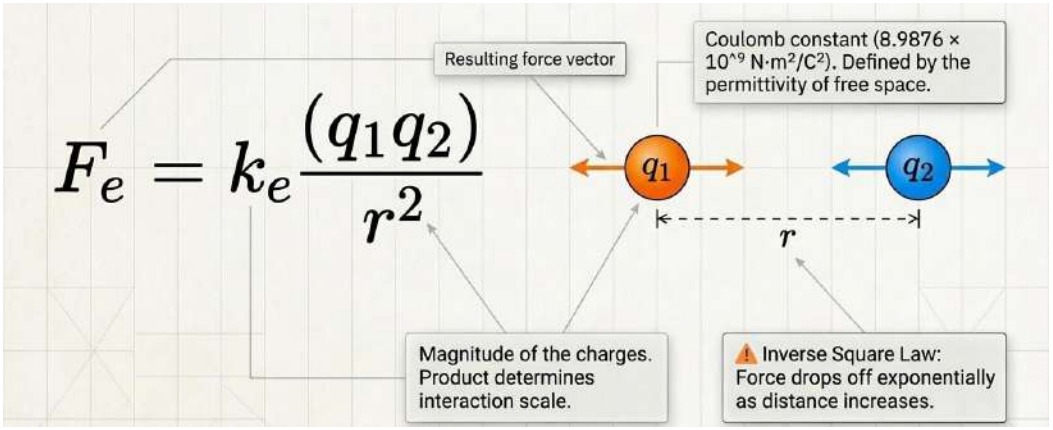
↑ Directly proportional to

$q_1 \times q_2$

↓ Inversely proportional to

r^2

Mathematical Formula



Newton's Third Law

The force on q_1 from q_2 is **equal in magnitude and opposite in direction** to the force on q_2 from q_1: $F_{12} = -F_{21}$

Calculate This

If distance doubles, what happens to force?

Section 8

Coulomb's Law - Worked Example

Problem Statement

Two point charges, $q_1 = +25$ nC and $q_2 = -75$ nC, are separated by a distance $r = 3.0$ cm. Find the magnitude and direction of the electric force that q_1 exerts on q_2.

Step 1: Convert Units

$q_1 = +25$ nC
$= +25 \times 10^{-9}$ C

$q_2 = -75$ nC
$= -75 \times 10^{-9}$ C

$r = 3.0$ cm
$= 0.030$ m

Step 2: Calculate

$F = k|q_1 q_2|/r^2$

$= (9.0 \times 10^9) \times |(25\times10^{-9})(-75\times10^{-9})| / (0.030)^2$

$= (9.0 \times 10^9) \times (1.875 \times 10^{-15}) / (9.0 \times 10^{-4})$

$F = 0.019$ N

$F = 19$ mN

Step 3: Direction

$q_1 = +$ and $q_2 = -$

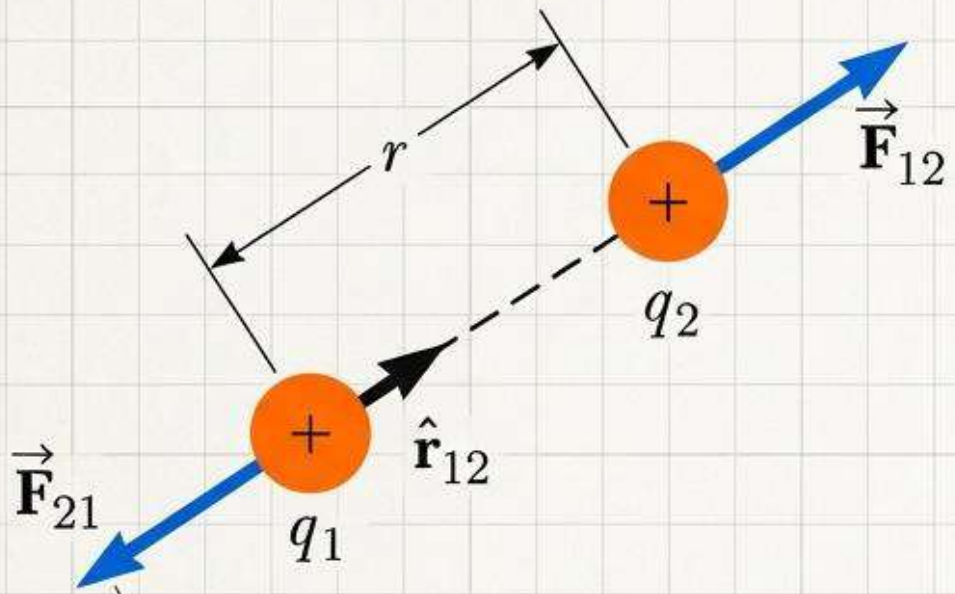
Opposite charges ATTRACT!

The force on q_2 is directed **toward q_1** (to the left if q_1 is on the right).

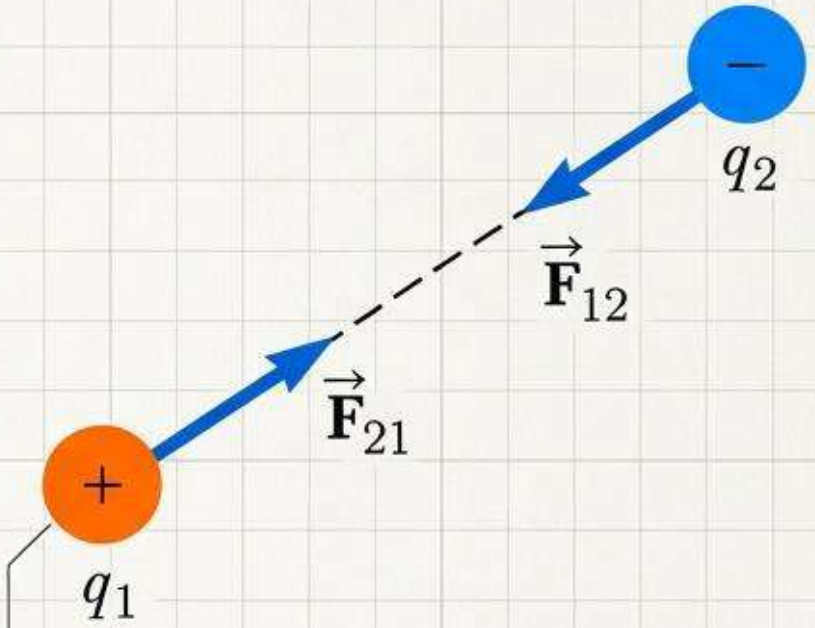
Final Answer

Magnitude: 0.019 N (19 mN) | Direction: Attractive (toward q_1)

The Vector Nature of Interaction



Repulsion: Like signs result in a positive product. The force pushes outward along the axis.

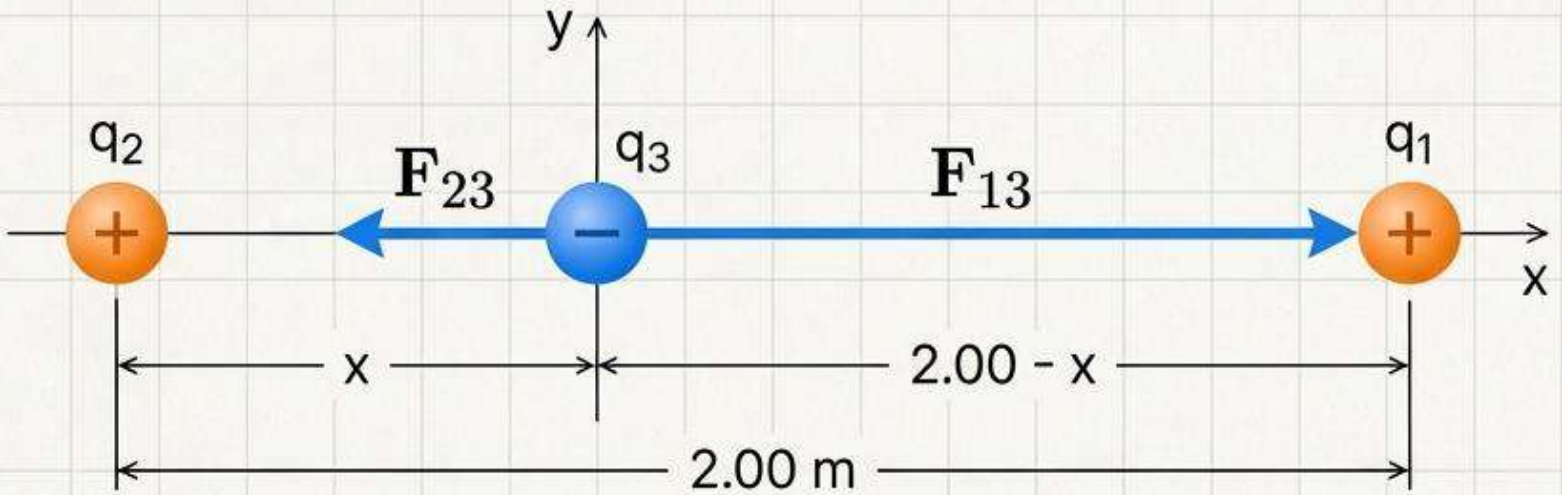


Attraction: Opposite signs result in a negative product. The force pulls inward along the axis.

$$F_{12} = -F_{21}$$

Newton's Third Law: Regardless of charge size, the force on q_1 is always equal in magnitude and opposite in direction to the force on q_2.

The Superposition Principle



$$\mathbf{F}_{\text{net}} = \mathbf{F}_{13} + \mathbf{F}_{23}$$

In a crowded space, the resultant force on any single charge is simply the vector sum of all individual forces acting upon it.

To find equilibrium (Zero Resultant Force), the magnitudes of opposing forces must be perfectly equal.

Section 9

The Electric Field Concept

The Problem

When two charged particles interact in empty space, **how does each one know the other is there?**

Solution: We use the concept of electric field as an intermediary!

Two-Stage Process

1. **Source Charge Creates Field**

   Charge Q modifies the properties of space around it, creating an electric field E.

2. **Test Charge Experiences Force**

   A test charge q_0 placed in the field experiences a force $F = q_0E$.

Analogy: Gravitational Field

Just as Earth creates a gravitational field g that exerts force on mass ($F = mg$), a charge creates an electric field E that exerts force on other charges ($F = qE$).

Electric Field Definition

$$E = F/q_0$$

E	= Electric field (N/C or V/m)
F	= Force on test charge (N)
q_0	= Test charge (C)

The electric field is the force per unit charge!

Force Direction

If $q_0 > 0$

F and E point in **same direction**

If $q_0 < 0$

F and E point in **opposite directions**

Quick Quiz

What is the unit of electric field?

Section 10

Electric Field of a Point Charge

Formula Derivation

Starting from Coulomb's Law:

$$F = k|q_0 q|/r^2$$

Using $E = F/q_0$:

$$E = k|q|/r^2$$

E	= Electric field magnitude (N/C)
k	= 8.988×10^9 N·m²/C²
q	= Source charge (C)
r	= Distance from charge (m)

Key Relationships

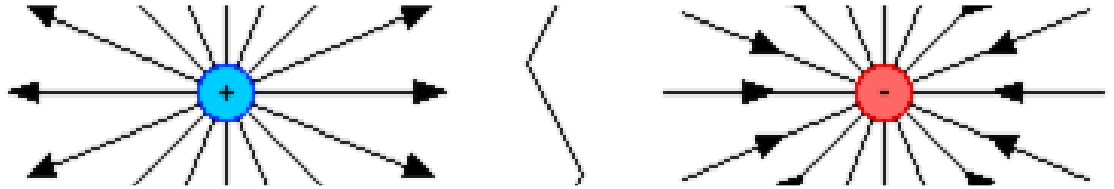
$E \propto |q|$
Stronger field from larger charge

$E \propto 1/r^2$
Field weakens with distance

Field Direction

Positive Charge
Field points **AWAY** from charge

Negative Charge
Field points **TOWARD** charge



Example Calculation

Problem: What is E at 2.0 m from q = 4.0 nC?

$E = k|q|/r^2$

$= (9.0 \times 10^9)(4.0 \times 10^{-9})/(2.0)^2$

$E = 9.0$ N/C

Think!

At what distance is E = 1 N/C from q = 1 μC?

Section 11

Electric Field Lines

What Are Field Lines?

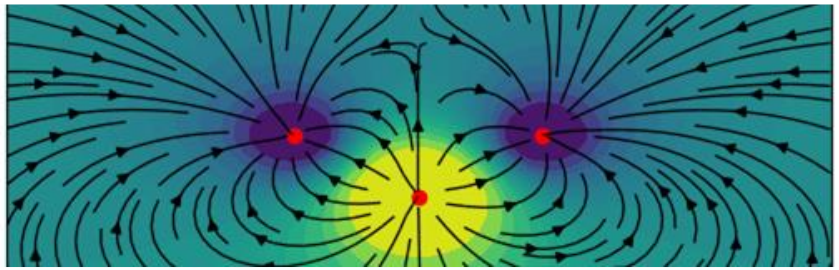
Electric field lines are **imaginary lines** used to visualize electric fields. They show both the **direction** and **strength** of the field at any point.

Field lines help us "see" the invisible electric field!

Properties of Field Lines

1. **Direction**
   Lines point **away** from positive charges and **toward** negative charges
2. **Origin and Termination**
   Lines begin on positive charges and end on negative charges
3. **Never Cross**
   Field lines **never intersect** each other (field has unique direction at each point)
4. **Density = Strength**
   Closer lines mean **stronger field**; farther apart means weaker field

Field Line Patterns



Field lines around multiple charges

Visual Interpretation

Tangent to Line
Gives field direction at that point

Line Density
Indicates field strength ($E \propto$ density)

Visual Quiz

Where is the field strongest?

Section 12

Superposition Principle

The Principle

The total electric field at any point is the vector sum of the fields due to each individual charge.

Mathematical Form

$$E_{total} = E_1 + E_2 + E_3 + \ldots$$

Important: This is a vector sum, not just adding magnitudes!

Key Insight

Each charge produces its field **independently**. The presence of other charges doesn't change the field each one produces.

Calculation Steps

1. **Calculate E from each charge**
   Use $E = k|q|/r^2$ for each
2. **Determine directions**
   Away from +, toward − charges
3. **Break into components**
   x and y components if needed
4. **Sum the components**
   Add x's and y's separately
5. **Find magnitude & direction**
   Use Pythagorean theorem and trig

Challenge

Two equal + charges. What is E at midpoint?

Section 13

Electric Dipole

Definition

An **electric dipole** consists of two point charges with **equal magnitude** and **opposite sign**, separated by a small distance.

+q and −q separated by distance d

Dipole Moment

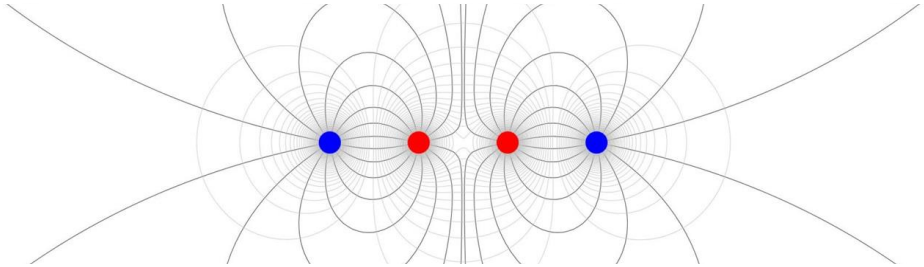
$$p = q \times d$$

p = dipole moment (C·m)
q = magnitude of charge (C)
d = separation distance (m)

Real-World Examples

- Water molecules (H_2O)
- Polar molecules in chemistry
- Antennas in communications

Dipole Field Pattern



Field lines go from + to −, creating a characteristic pattern

Field Characteristics

Along the Axis
Field is parallel to dipole moment, strongest near charges

Perpendicular Bisector
Field is opposite to dipole moment, weaker than on axis

Far Field (r » d)
$E \propto 1/r^3$ (falls off faster than single charge)

Final Question

Why does dipole field fall off as $1/r^3$?

Summary & Key Takeaways

1 Two Types of Charge

Positive (+) and **Negative (–)**. Like charges repel, opposite charges attract.

2 Conservation of Charge

Total charge in a closed system is **constant**. Charge is only transferred, not created.

3 Conductors vs Insulators

Conductors allow charge flow (metals); **Insulators** resist it (rubber, glass).

4 Coulomb's Law

$F = k|q_1 q_2|/r^2$ — Force is proportional to charges, inversely proportional to distance squared.

5 Electric Field

$E = F/q_0$ — Force per unit charge. Measured in N/C or V/m.

6 Field of Point Charge

$E = k|q|/r^2$ — Points away from + charges, toward – charges.

7 Superposition Principle

Total field is the **vector sum** of fields from all charges: $E_{total} = E_1 + E_2 + \ldots$

8 Electric Dipole

Two equal and opposite charges. Creates characteristic field pattern with moment $p = qd$.

Practice Problems & Ask Questions!

Electric Field – Introduction

- The electric force is a field force.
- Field forces can act through space.
  - The effect is produced even with no physical contact between objects.
- Faraday developed the concept of a field in terms of electric fields.

Electric Field – Definition

- An **electric field** is said to exist in the region of space around a charged object.
  - This charged object is the **source charge.**
- When another charged object, the **test charge**, enters this electric field, an electric force acts on it.

Electric Field – Definition, cont

- The electric field is defined as the electric force on the test charge per unit charge.
- The electric field vector, $\vec{\mathbf{E}}$, at a point in space is defined as the electric force acting on a positive test charge, q_o, placed at that point divided by the test charge:

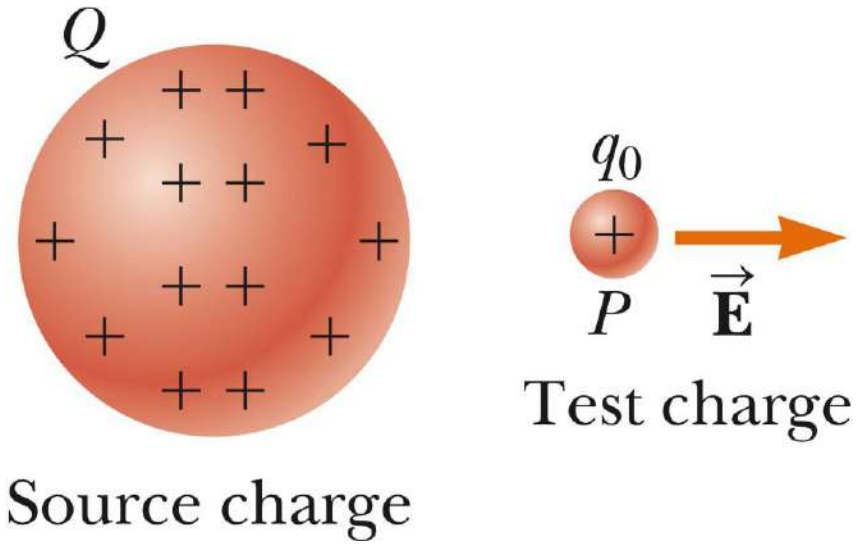
$$\vec{\mathbf{E}} \equiv \frac{\vec{\mathbf{F}}}{q_o}$$

Electric Field, Notes

- $\vec{\mathbf{E}}$ is the field produced by some charge or charge distribution, separate from the test charge.
- The existence of an electric field is a property of the source charge.
  - The presence of the test charge is not necessary for the field to exist.
- The test charge serves as a detector of the field.

Electric Field Notes, Final

- The direction of $\vec{\mathbf{E}}$ is that of the force on a positive test charge.
- The SI units of $\vec{\mathbf{E}}$ are N/C.
- We can also say that an electric field exists at a point if a test charge at that point experiences an electric force.



Relationship Between F and E

- $\vec{\mathbf{F}}_e = q\vec{\mathbf{E}}$
  - This is valid for a point charge only.
  - One of zero size
  - For larger objects, the field may vary over the size of the object.
- If *q* is positive, the force and the field are in the same direction.
- If *q* is negative, the force and the field are in opposite directions.

Electric Field, Vector Form

- Remember Coulomb's law, between the source and test charges, can be expressed as

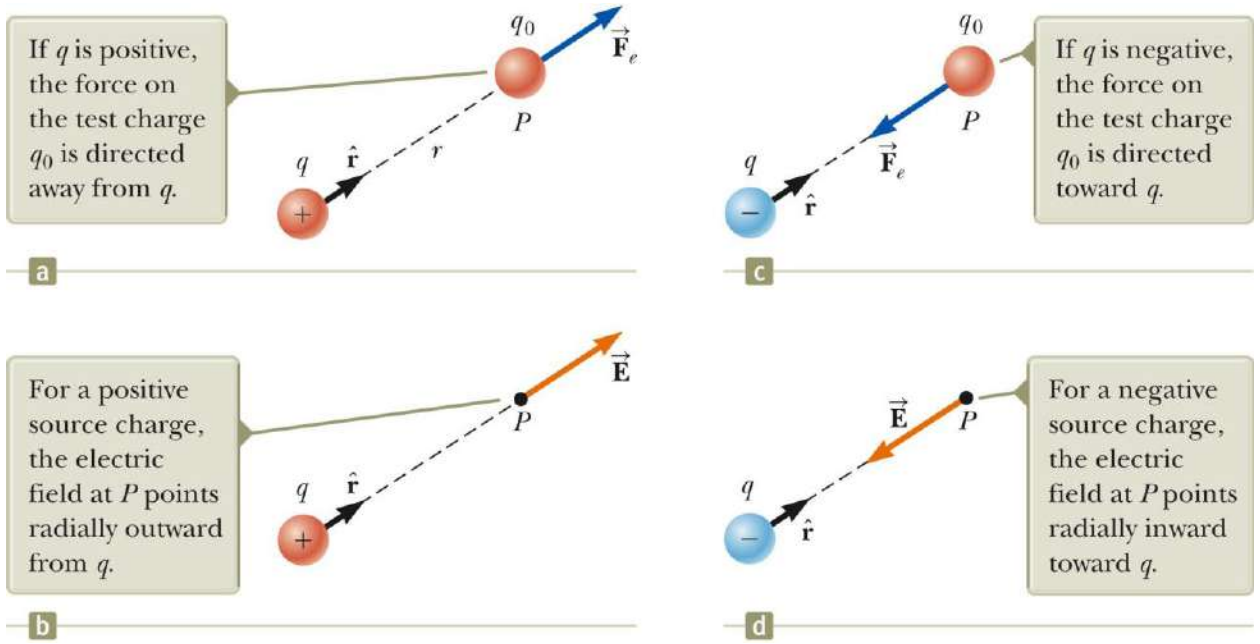
$$\vec{\mathbf{F}}_e = k_e \frac{q q_o}{r^2} \hat{\mathbf{r}}$$

- Then, the electric field will be

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}_e}{q_o} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

More About Electric Field Direction

- a) *q* is positive, the force is directed away from *q.*
- b) The direction of the field is also away from the positive source charge.
- c) *q* is negative, the force is directed toward *q.*
- d) The field is also toward the negative source charge.



Electric Fields from Multiple Charges

- At any point *P*, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges.

$$\vec{\mathbf{E}} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

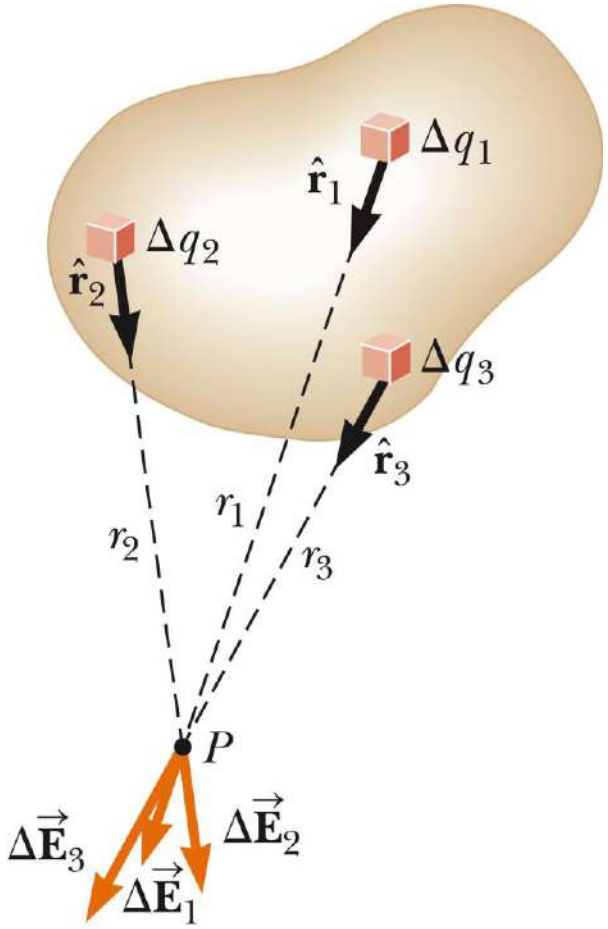
Electric Field – Continuous Charge Distribution

- The distances between charges in a group of charges may be much smaller than the distance between the group and a point of interest.
- In this situation, the system of charges can be modeled as continuous.
- The system of closely spaced charges is equivalent to a total charge that is continuously distributed along some line, over some surface, or throughout some volume.

Electric Field – Continuous Charge Distribution, cont

•Procedure:

- Divide the charge distribution into small elements, each of which contains Δ*q.*
- Calculate the electric field due to one of these elements at point *P.*
- Evaluate the total field by summing the contributions of all the charge elements.



Electric Field – Continuous Charge Distribution, equations

- For the individual charge elements

$$\Delta\vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}}$$

- Because the charge distribution is continuous

$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}$$

Charge Densities

- **Volume charge density**: when a charge is distributed evenly throughout a volume
  - $\rho \equiv Q / V$ with units C/m^3
- **Surface charge density**: when a charge is distributed evenly over a surface area
  - $\sigma \equiv Q / A$ with units C/m^2
- **Linear charge density**: when a charge is distributed along a line
  - $\lambda \equiv Q / \ell$ with units C/m

Amount of Charge in a Small Volume

- If the charge is nonuniformly distributed over a volume, surface, or line, the amount of charge, dq, is given by
  - For the volume: $dq = \rho\, dV$
  - For the surface: $dq = \sigma\, dA$
  - For the length element: $dq = \lambda\, d\ell$

Problem-Solving Strategy

- *Conceptualize*
  - Establish a mental representation of the problem.
  - Image the electric field produced by the charges or charge distribution.
- *Categorize*
  - Individual charge?
  - Group of individual charges?
  - Continuous distribution of charges?

Problem-Solving Strategy, cont

- *Analyze*
  - **Analyzing a group of individual charges**:
    - Use the superposition principle, find the fields due to the individual charges at the point of interest and then add them as vectors to find the resultant field.
    - Be careful with the manipulation of vector quantities.
  - **Analyzing a continuous charge distribution:**
    - The vector sums for evaluating the total electric field at some point must be replaced with vector integrals.
    - Divide the charge distribution into infinitesimal pieces, calculate the vector sum by integrating over the entire charge distribution.
  - **Symmetry:**
    - Take advantage of any symmetry to simplify calculations.

Problem Solving Hints, final

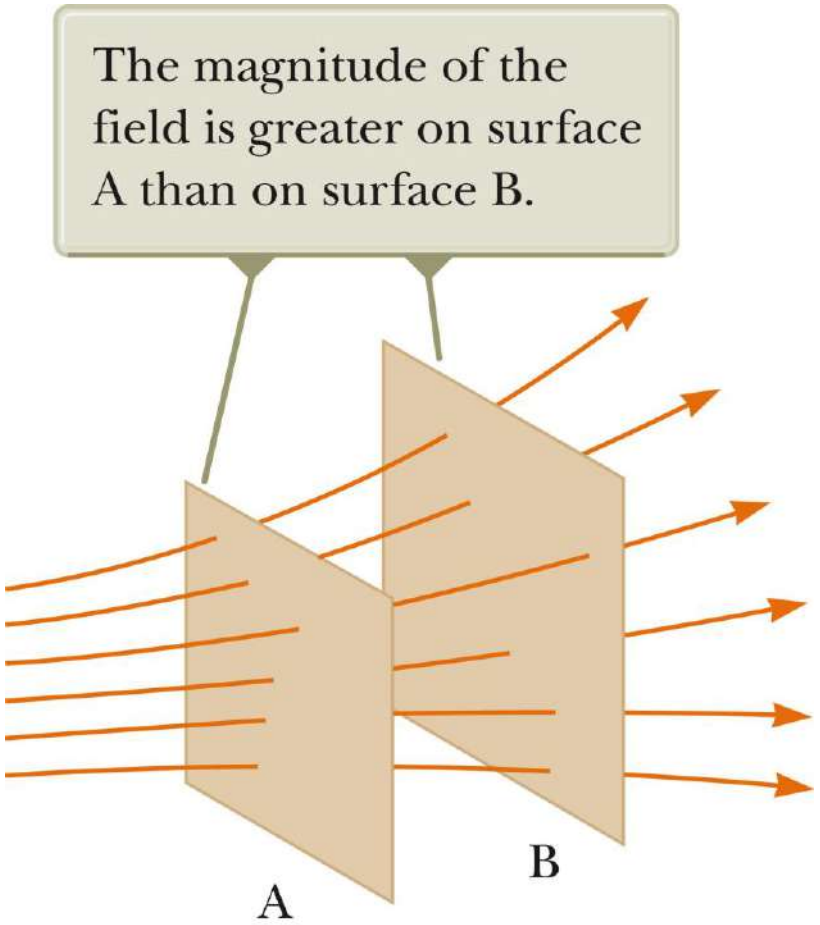
- *Finalize*
  - Check to see if the electric field expression is consistent with your mental representation.
  - Check to see if the solution reflects any symmetry present.
  - Image varying parameters to see if the mathematical result changes in a reasonable way.

Electric Field Lines

- Field lines give us a means of representing the electric field pictorially.
- The electric field vector is tangent to the electric field line at each point.
  - The line has a direction that is the same as that of the electric field vector.
- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region.

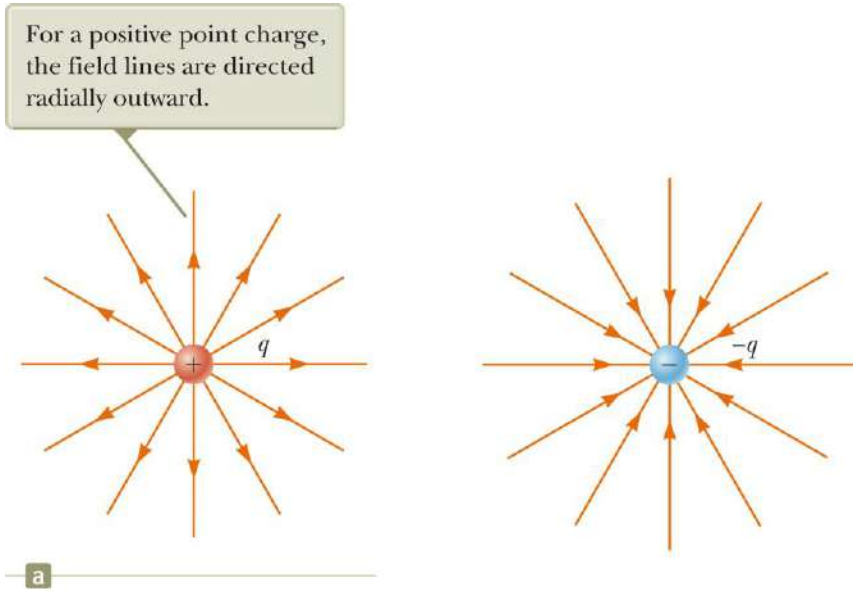
Electric Field Lines, General

- The density of lines through surface A is greater than through surface B.
- The magnitude of the electric field is greater on surface A than B.
- The lines at different locations point in different directions.
  - This indicates the field is nonuniform.



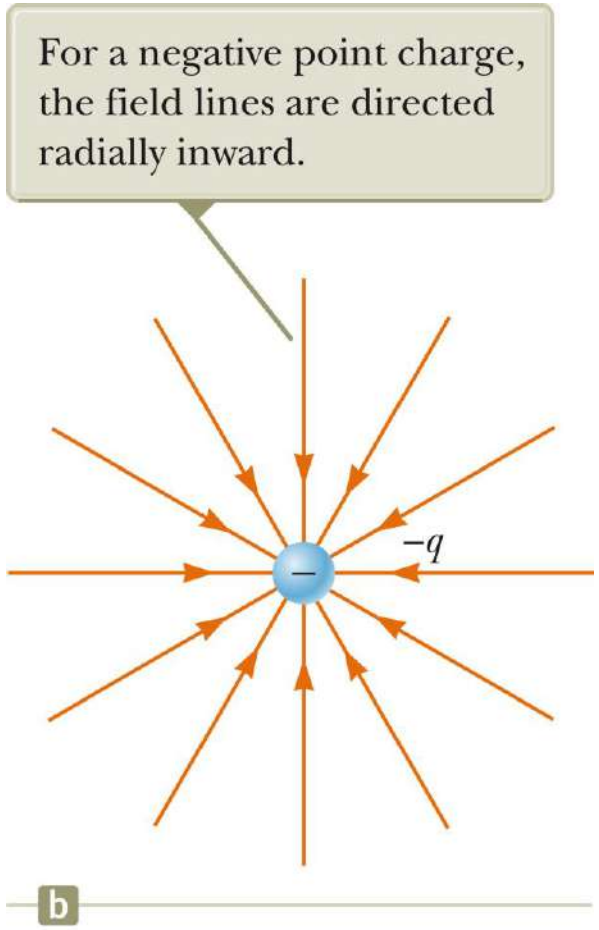
Electric Field Lines, Positive Point Charge

- The field lines radiate outward in all directions.
  - In three dimensions, the distribution is spherical.
- The lines are directed away from the source charge.
  - A positive test charge would be repelled away from the positive source charge.



Electric Field Lines, Negative Point Charge

- The field lines radiate inward in all directions.
- The lines are directed toward the source charge.
  - A positive test charge would be attracted toward the negative source charge.

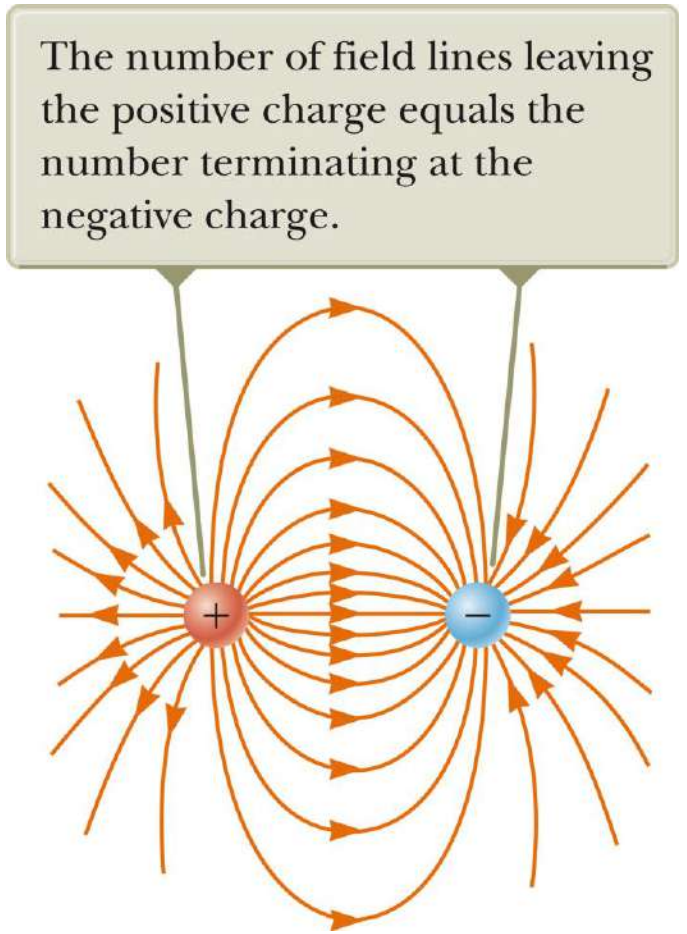


Electric Field Lines – Rules for Drawing

- The lines must begin on a positive charge and terminate on a negative charge.
  - In the case of an excess of one type of charge, some lines will begin or end infinitely far away.
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No two field lines can cross.
- Remember field lines are **not** material objects, they are a pictorial representation used to qualitatively describe the electric field.

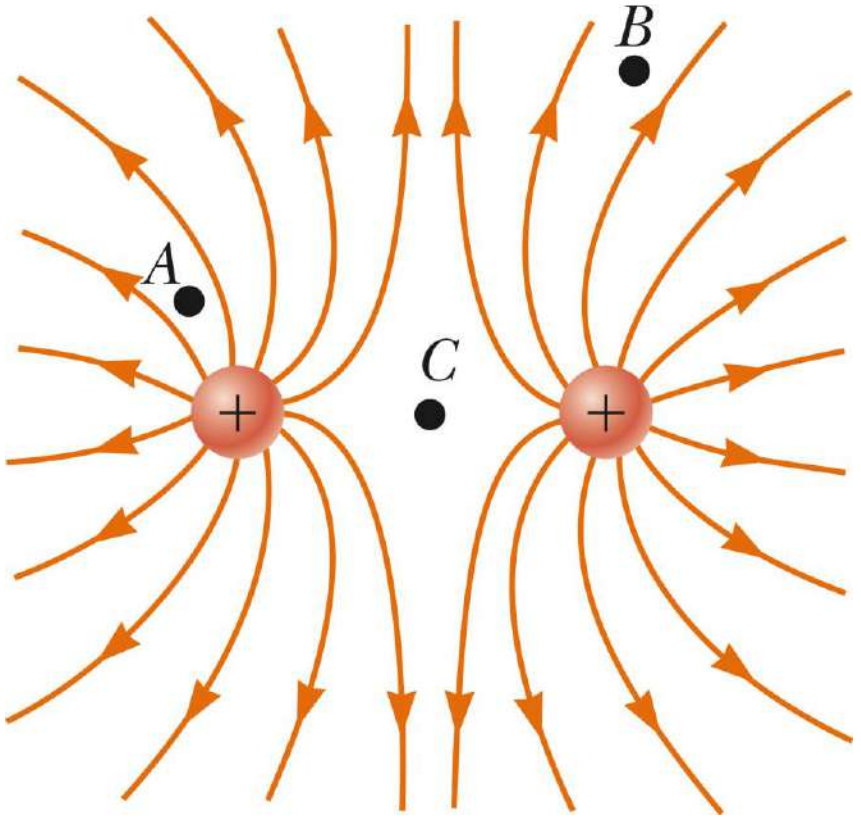
Electric Field Lines – Dipole

- The charges are equal and opposite.
- The number of field lines leaving the positive charge equals the number of lines terminating on the negative charge.



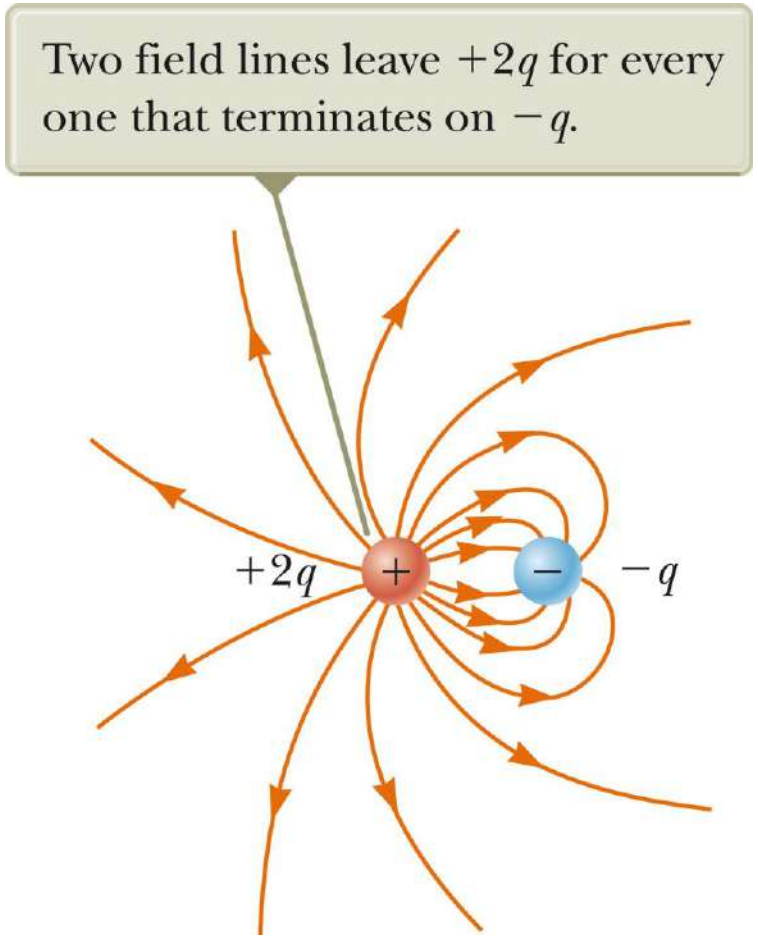
Electric Field Lines – Like Charges

- The charges are equal and positive.
- The same number of lines leave each charge since they are equal in magnitude.
- At a great distance, the field is approximately equal to that of a single charge of $2q$.
- Since there are no negative charges available, the field lines end infinitely far away.



Electric Field Lines, Unequal Charges

- The positive charge is twice the magnitude of the negative charge.
- Two lines leave the positive charge for each line that terminates on the negative charge.
- At a great distance, the field would be approximately the same as that due to a single charge of +*q*.



Motion of Charged Particles

- When a charged particle is placed in an electric field, it experiences an electrical force.
- If this is the only force on the particle, it must be the net force.
- The net force will cause the particle to accelerate according to Newton's second law.

Motion of Particles, cont

- $\vec{\mathbf{F}}_e = q\vec{\mathbf{E}} = m\vec{\mathbf{a}}$

•If the field is uniform, then the acceleration is constant.

•The particle under constant acceleration model can be applied to the motion of the particle.

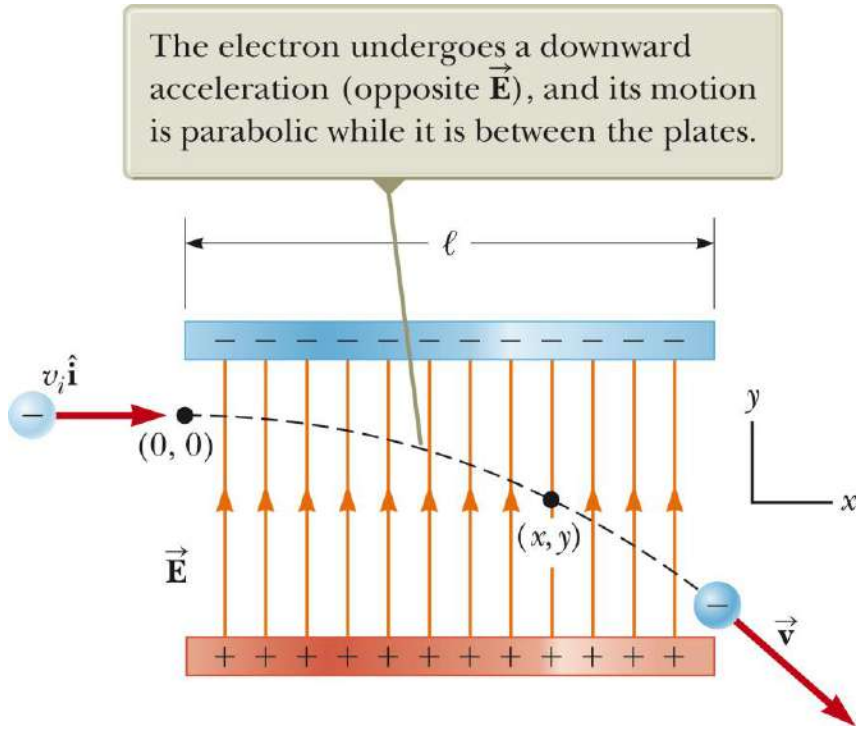
  - The electric force causes a particle to move according to the models of forces and motion.

•If the particle has a positive charge, its acceleration is in the direction of the field.

•If the particle has a negative charge, its acceleration is in the direction opposite the electric field.

Electron in a Uniform Field, Example

- The electron is projected horizontally into a uniform electric field.
- The electron undergoes a downward acceleration.
  - It is negative, so the acceleration is opposite the direction of the field.
- Its motion is parabolic while between the plates.



Electricity & Magnetism

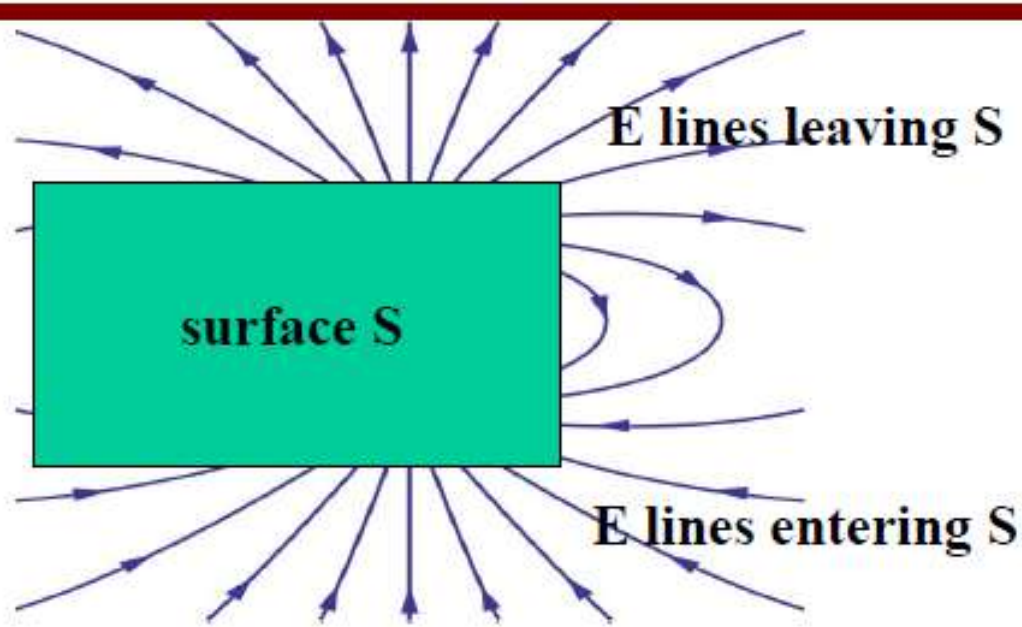
Gauss's Law

Gauss's Law

- **Gauss's Law gives us an *alternative* to Coulomb's Law for calculating the electric field due to a given distribution of charges.**

Gauss's Law: The General Idea

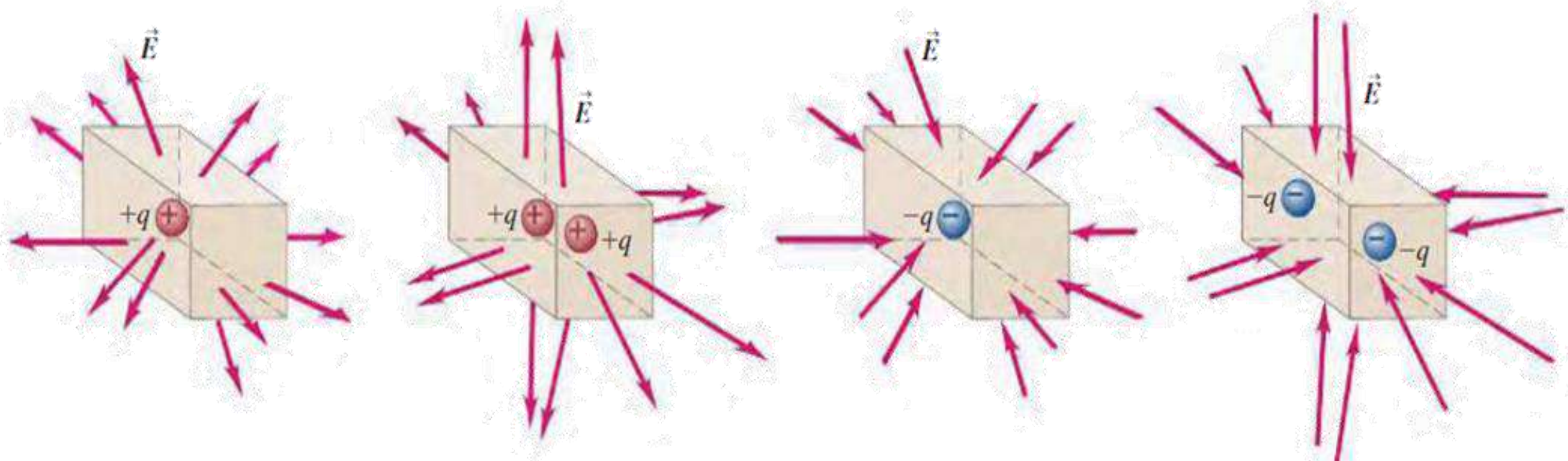
The net number of electric field lines which leave any surface of space is proportional to the net electric charge in that surface



Flux

The word “flux” comes from a Latin word meaning “flow.” Φ)

22.2 The electric field on the surface of boxes containing (a) a single positive point charge, (b) two positive point charges, (c) a single negative point charge, or (d) two negative point charges.



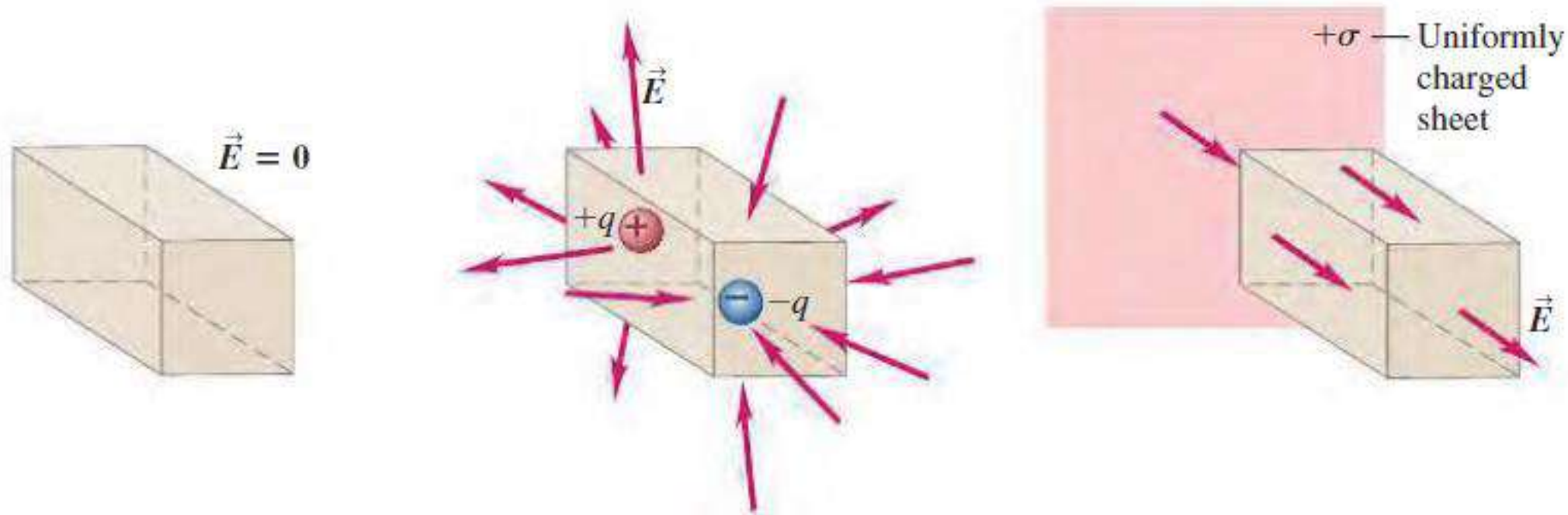
Flux

22.3 Three cases in which there is zero *net* charge inside a box and no net electric flux through the surface of the box. (a) An empty box with $\vec{E} = 0$. (b) A box containing one positive and one equal-magnitude negative point charge. (c) An empty box immersed in a uniform electric field.

(a) No charge inside box, zero flux

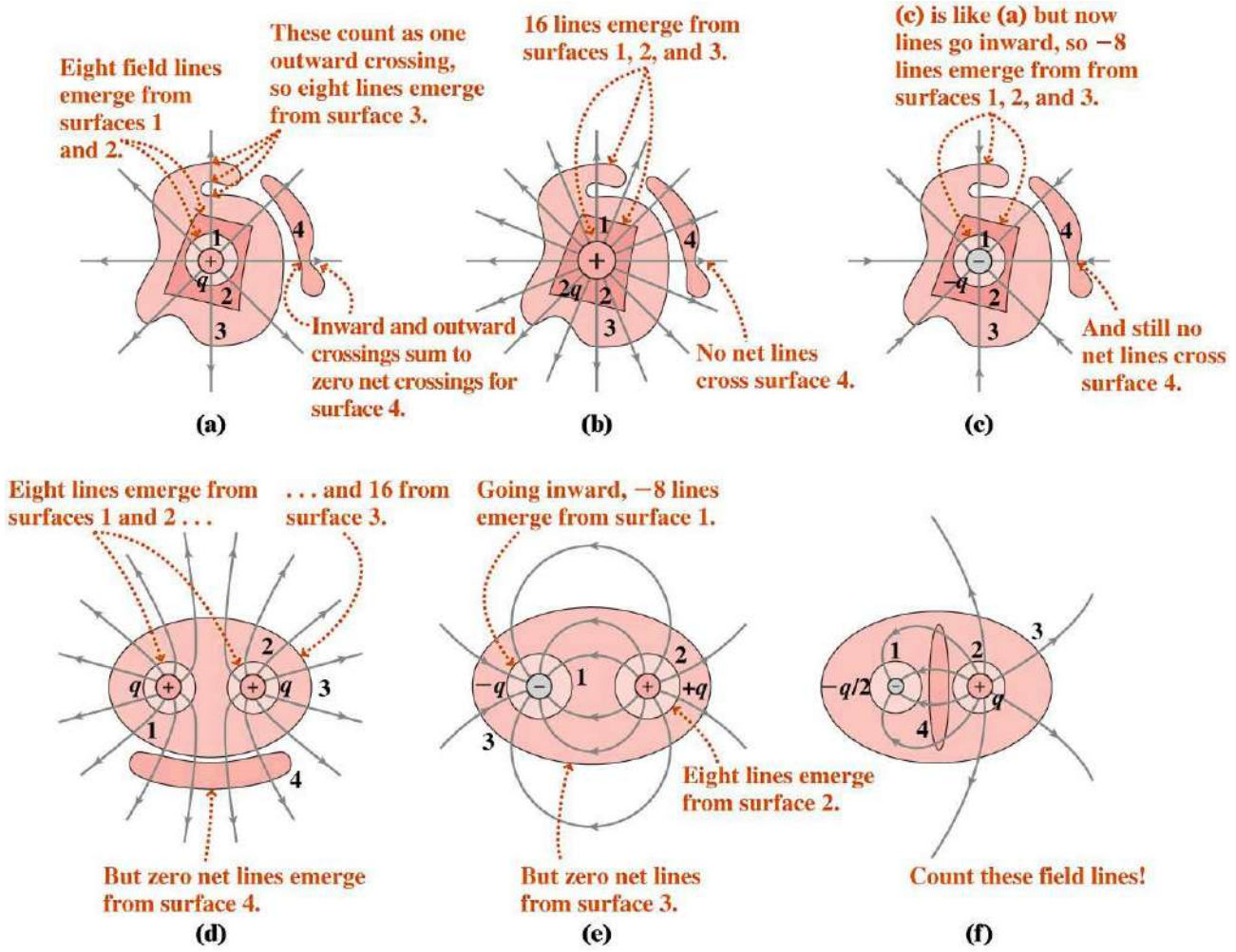
(b) Zero *net* charge inside box, inward flux cancels outward flux.

(c) No charge inside box, inward flux cancels outward flux.



- How many field lines emerge from closed surfaces surrounding charge?

- Count each field line crossing going outward as +1, each inward crossing as –1.
- You'll find that **the number of field lines crossing any closed surface is proportional to the net charge enclosed.**



Flux

The ***flux Φ*** of the ***field E*** through the ***surface S*** is **defined** as

$$\Phi = \int_S \vec{E} \cdot d\vec{A}$$

The **meaning** of flux is just the **number of field lines** passing through the surface.

Best Statement of Gauss's Law

The outward flux of the electric field through any closed surface equals the net enclosed charge divided by ε_0.

Gauss's Law: The Equation

$$\oint_S \vec{E} \cdot d\vec{A} = Q_{enc} / \varepsilon_0$$

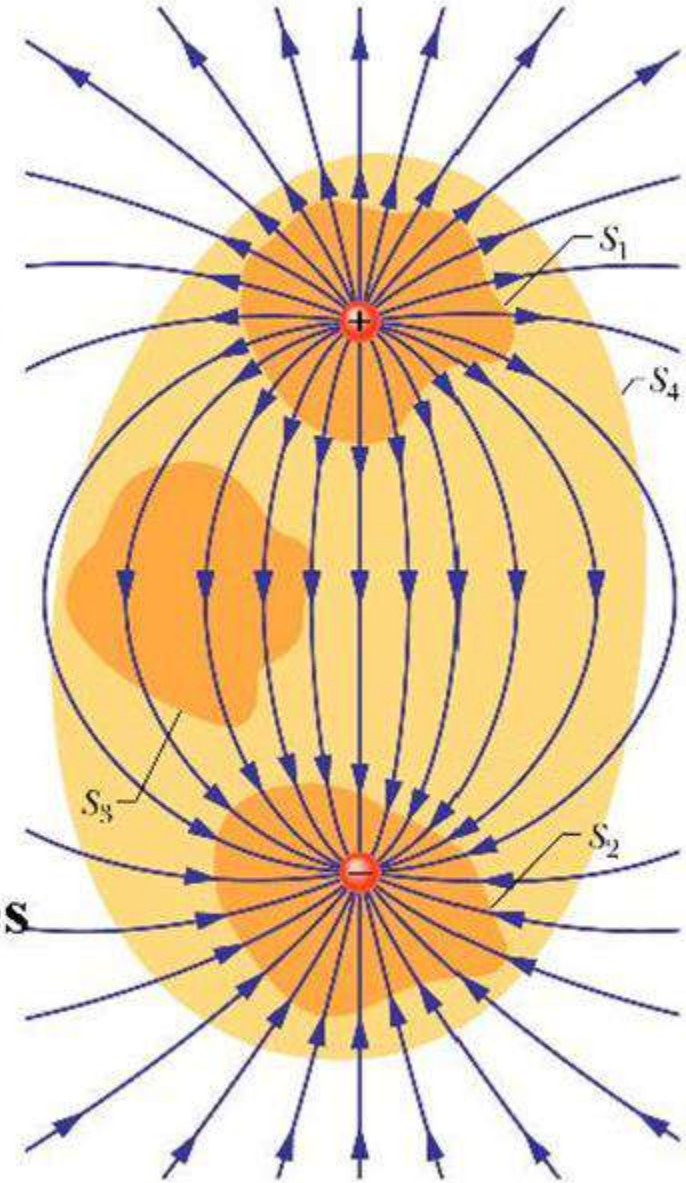
- S is any **closed** surface.
- Q_{enc} is the **net** charge **enclosed** within S.
- dA is an element of area on the surface of S.
- $\vec{dA}$ is in the direction of the **outward normal.**

$$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N.m^2$$

Flux Examples

Assume two charges, +q and –q.

.Find fluxes through surfaces



Remember flux is negative if lines are entering closed surface

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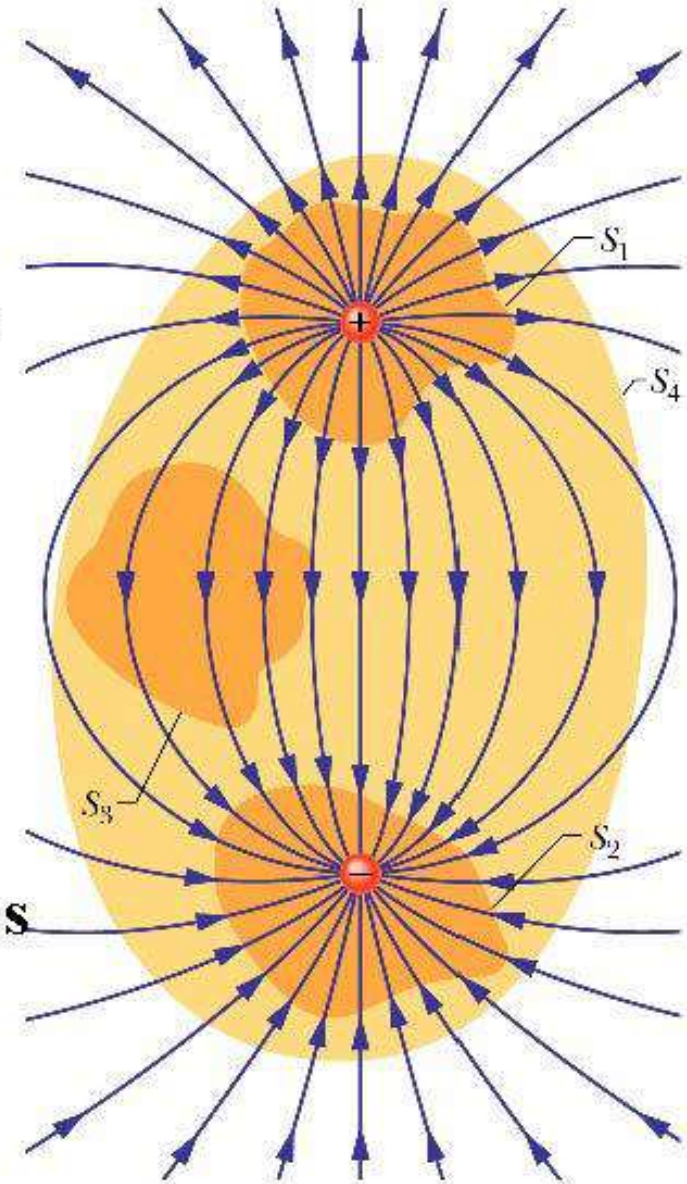
$\Phi_1 = +q/\varepsilon_0$

$\Phi_2 = -q/\varepsilon_0$

$\Phi_3 = 0$

$\Phi_4 = (q - q)/\varepsilon_0 = 0$

Remember flux is negative if lines are entering closed surface

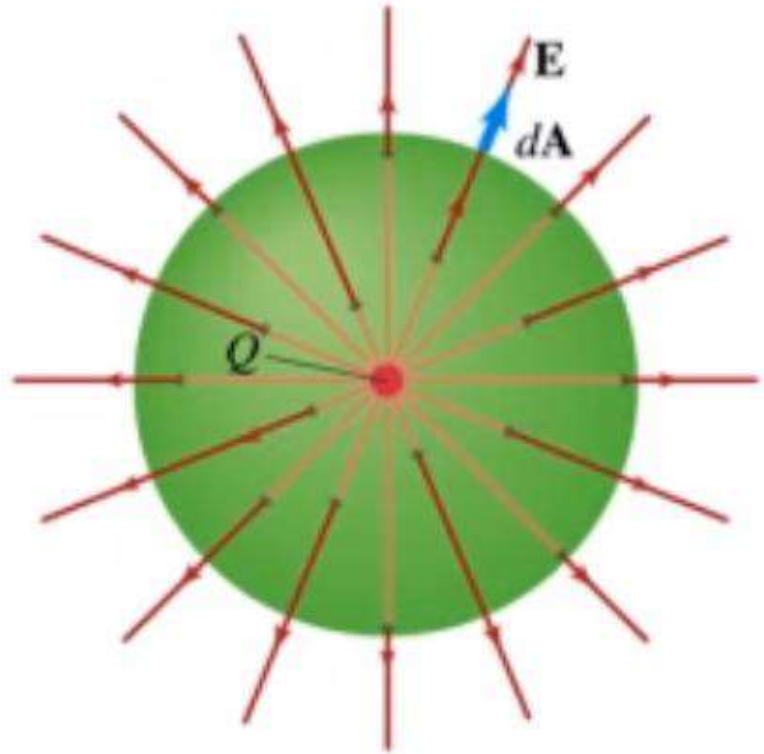


Gauss's Law and Coulomb's Law

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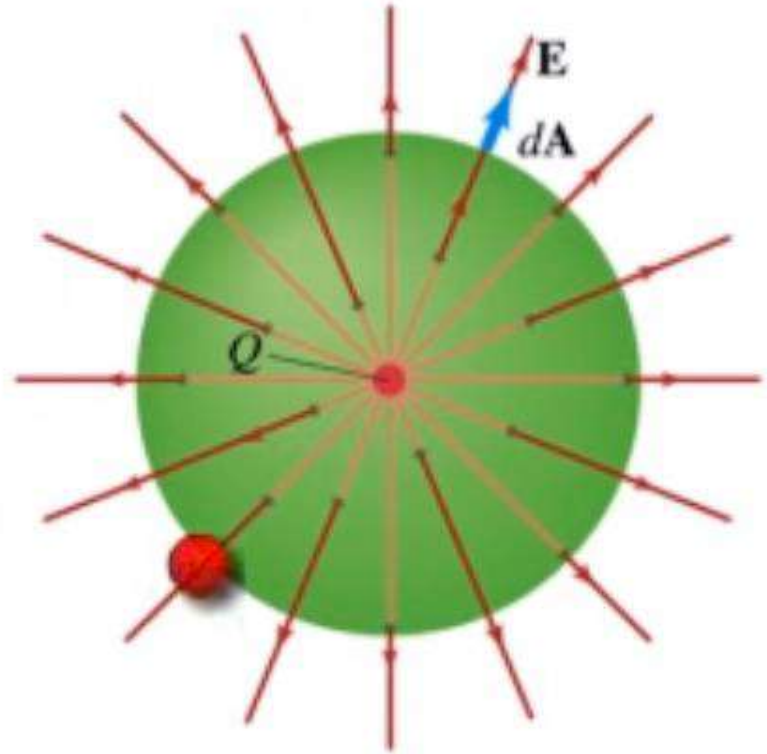
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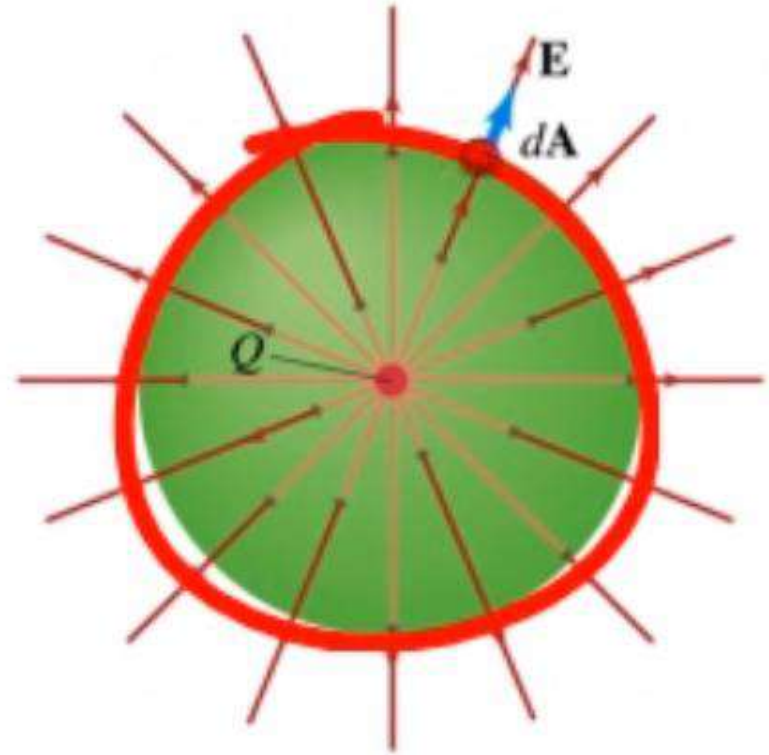
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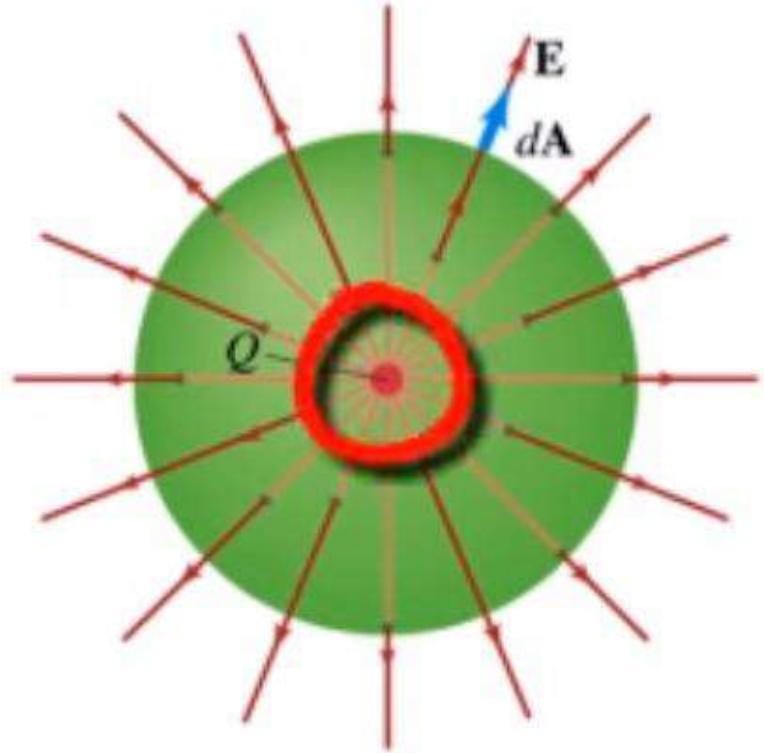
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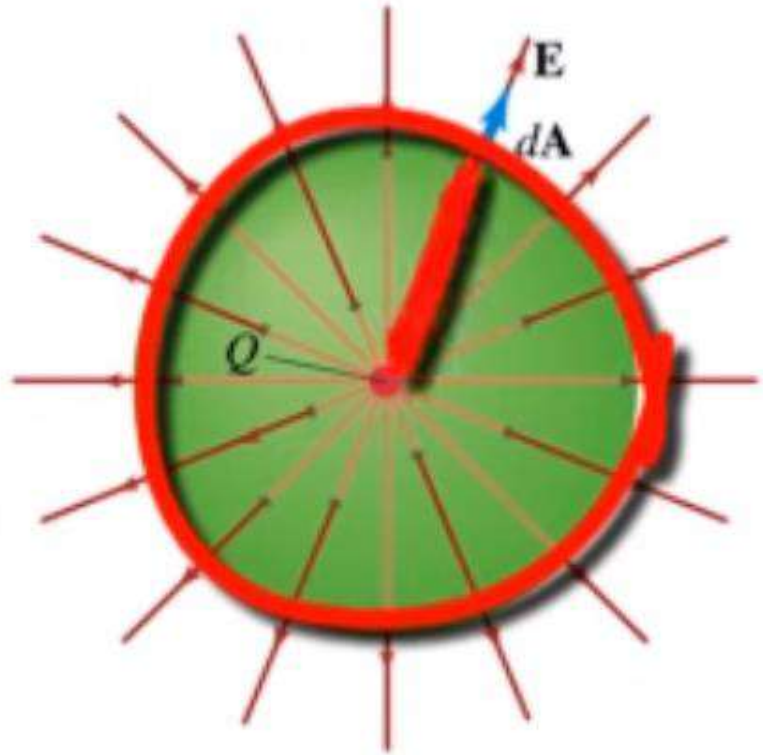
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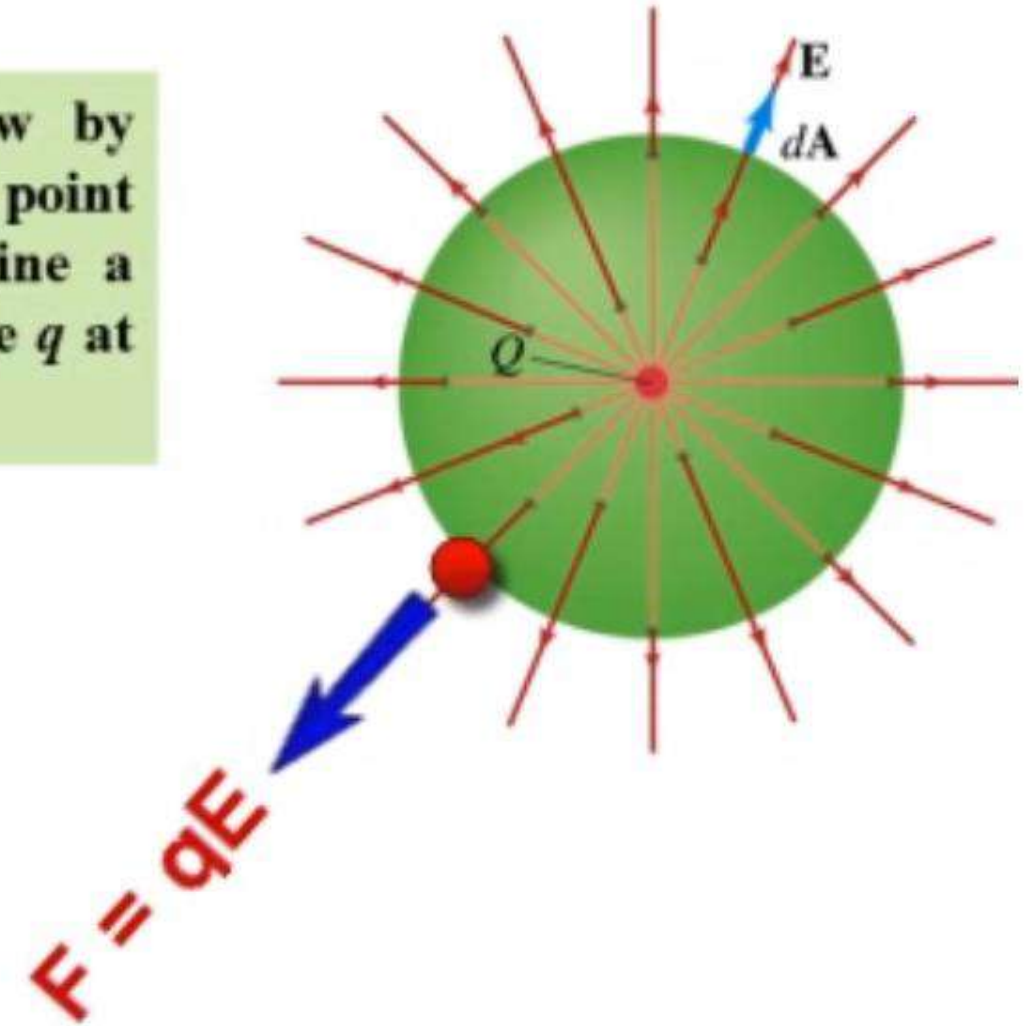
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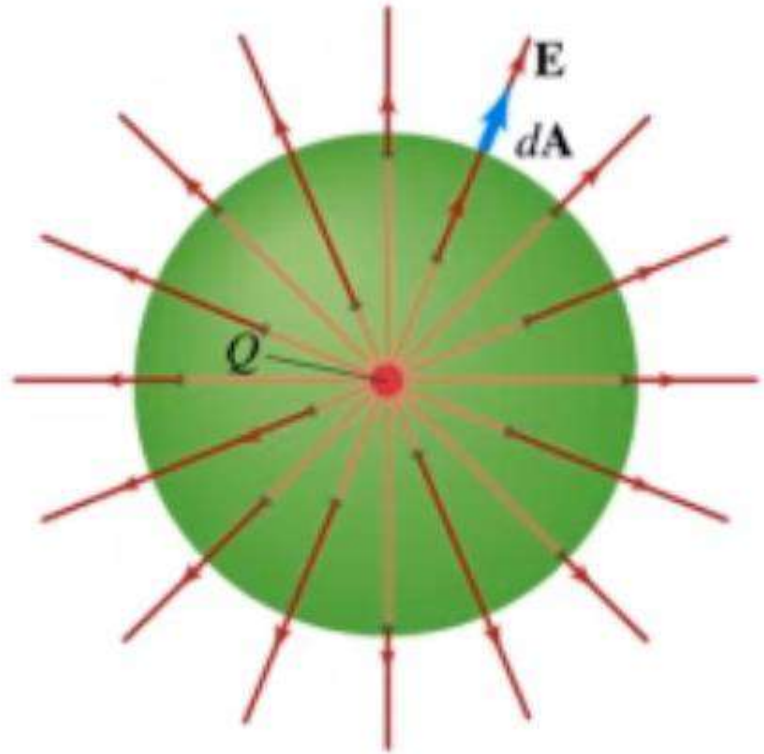
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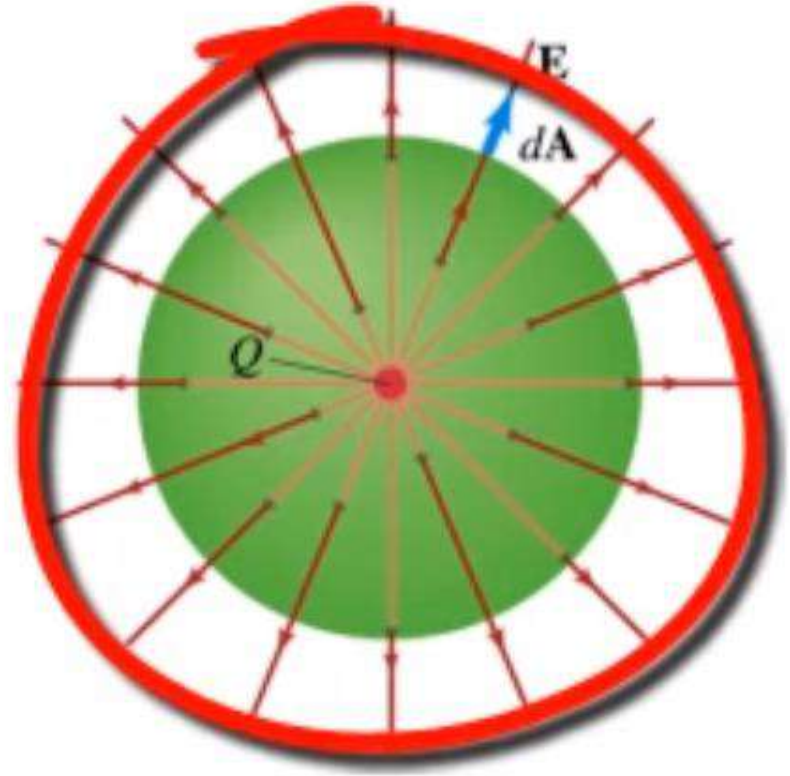
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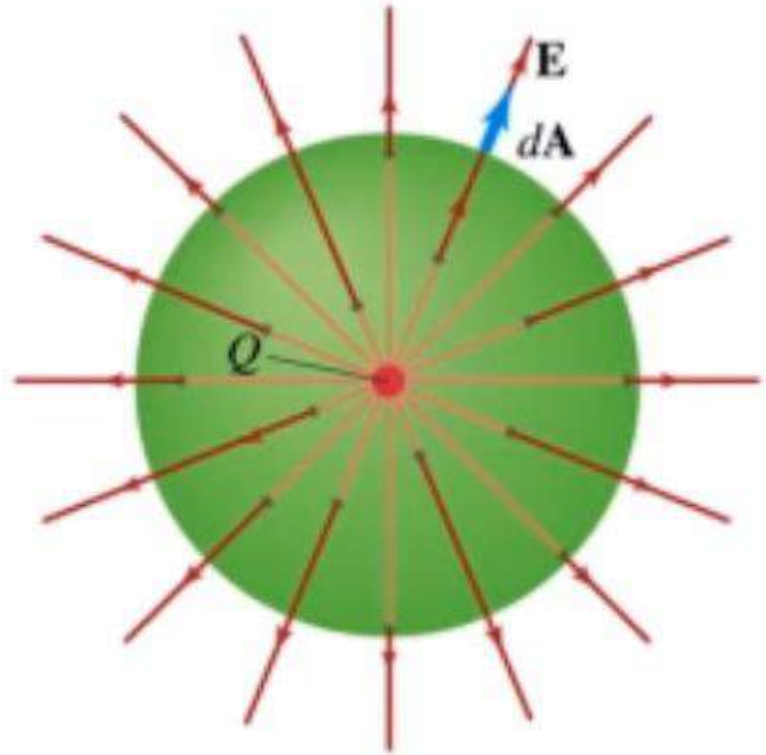
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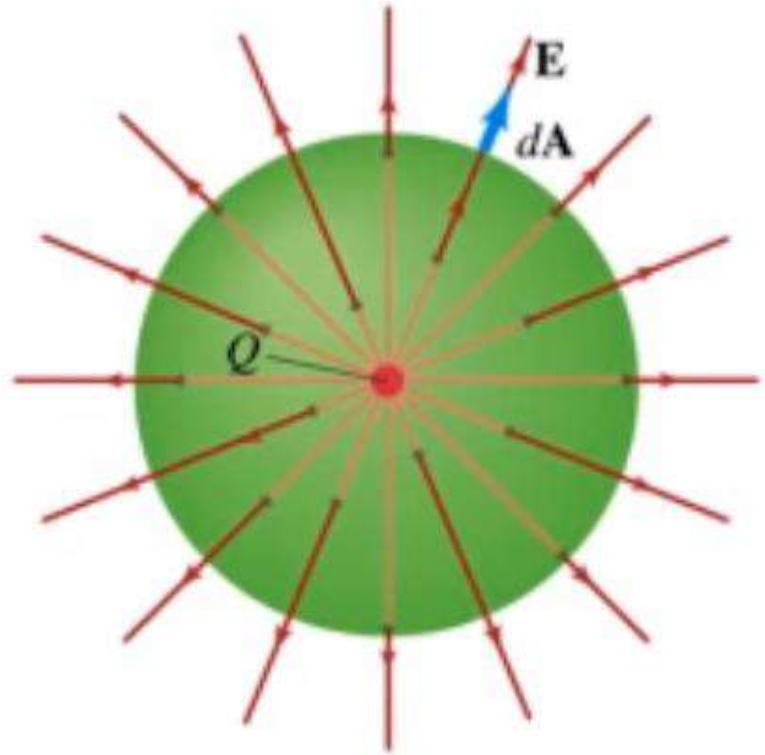
$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$



Gauss's Law and Coulomb's Law

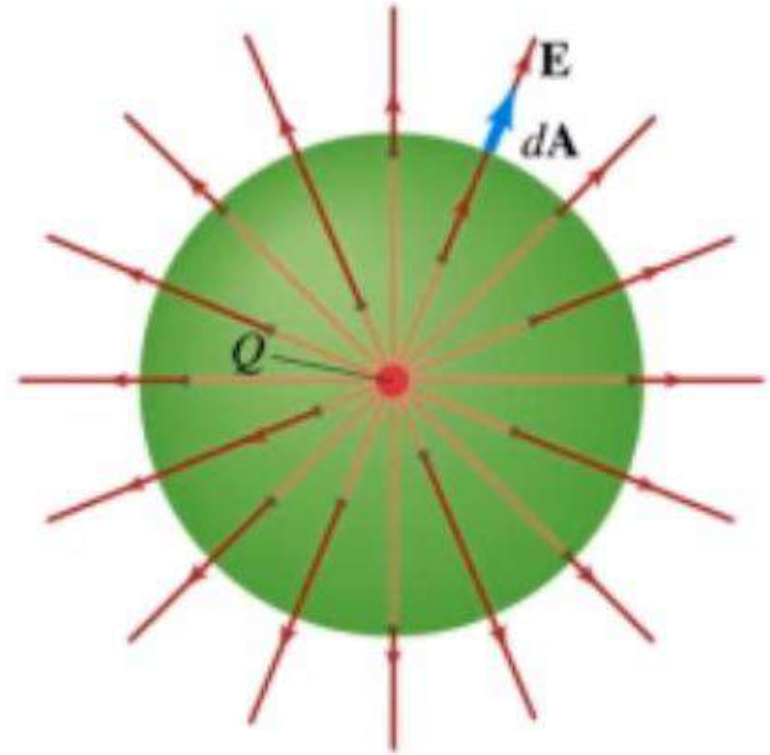
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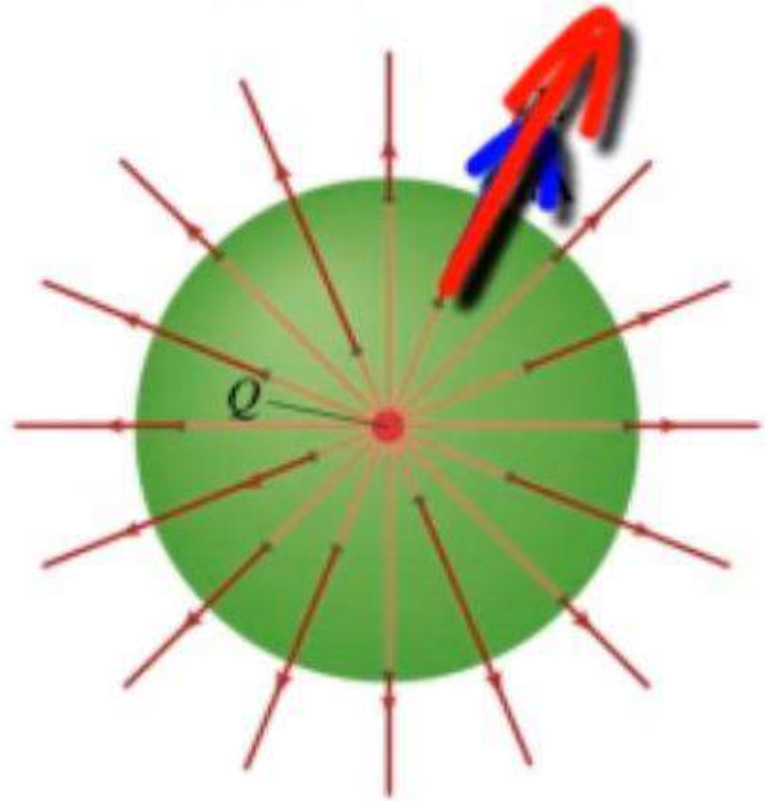


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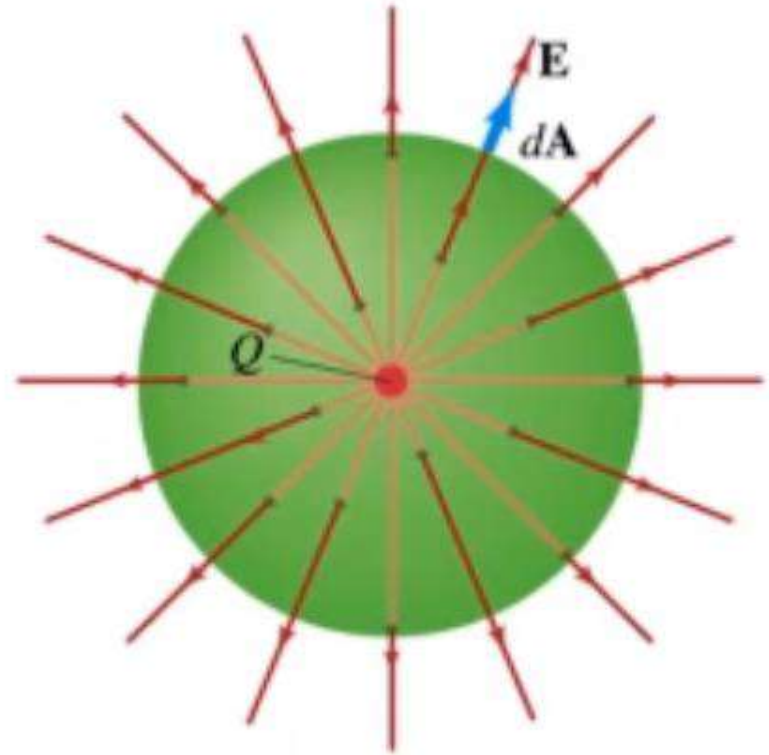


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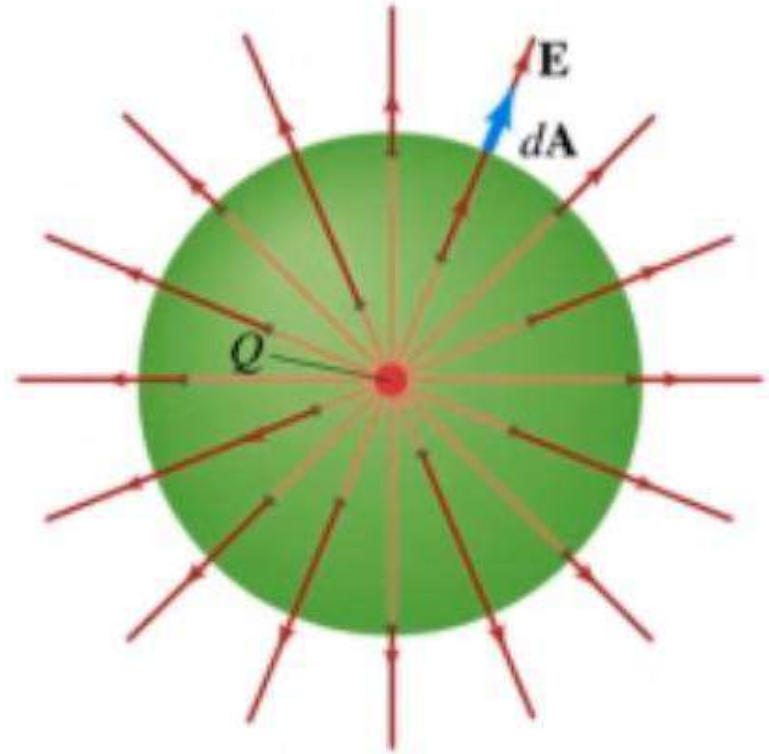


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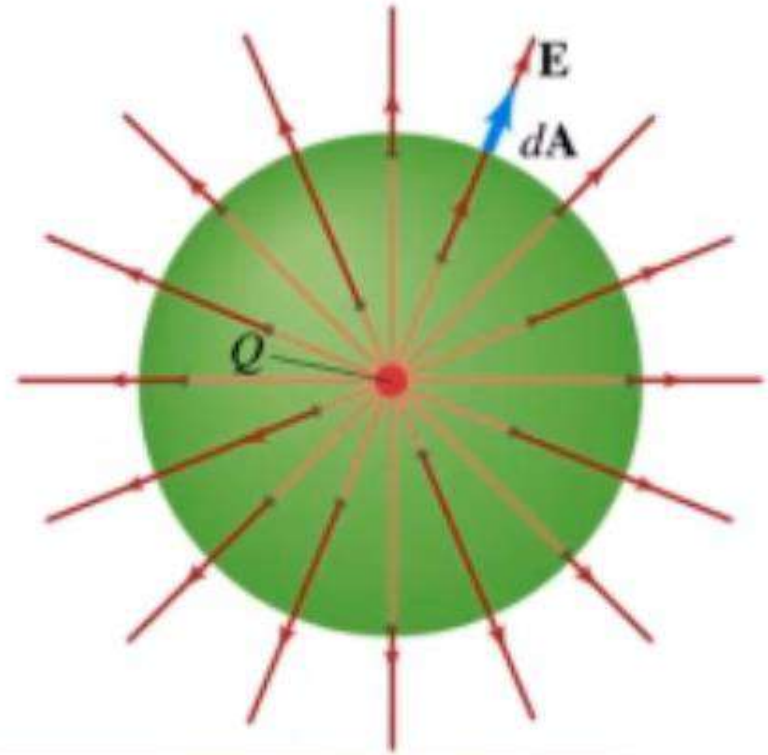


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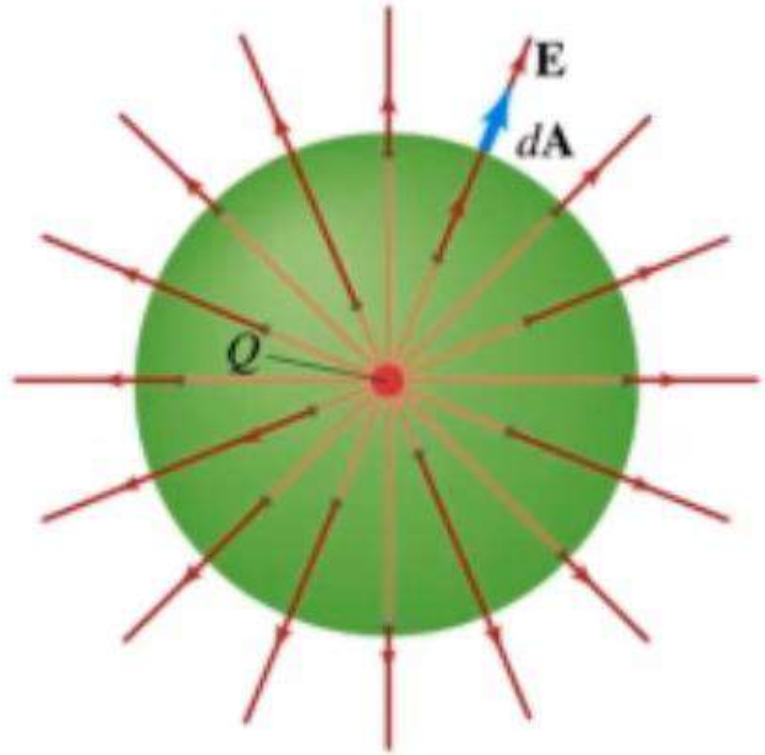
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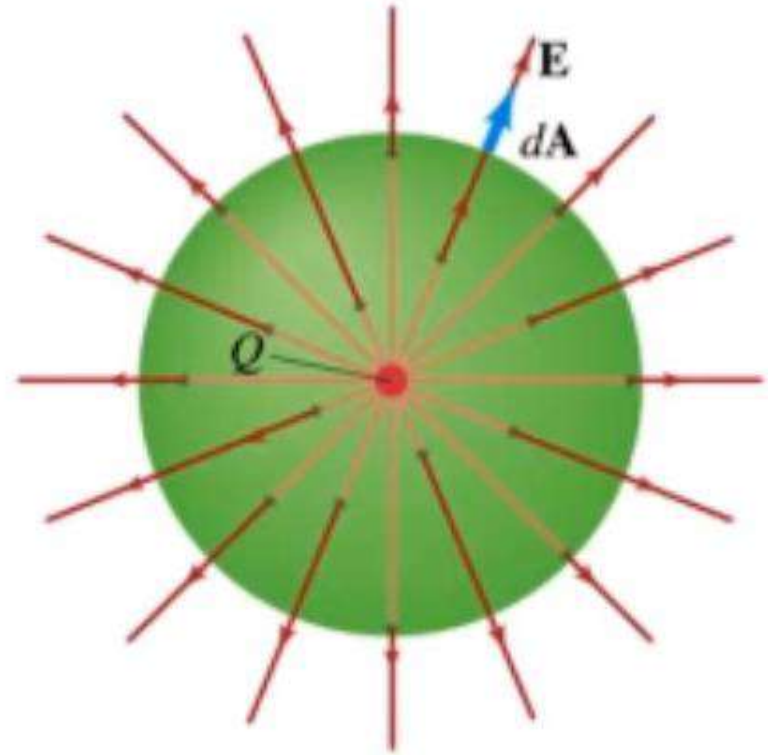
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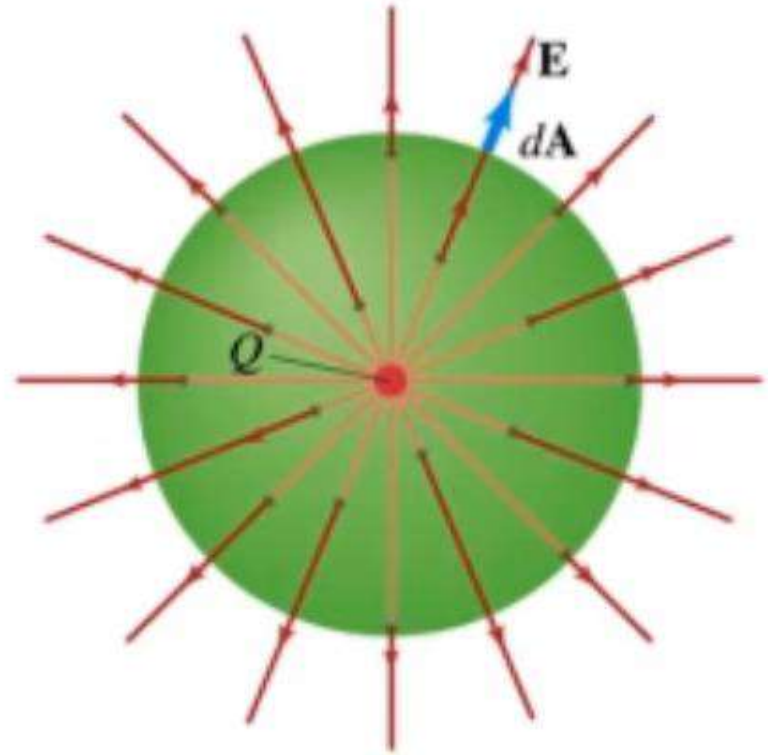
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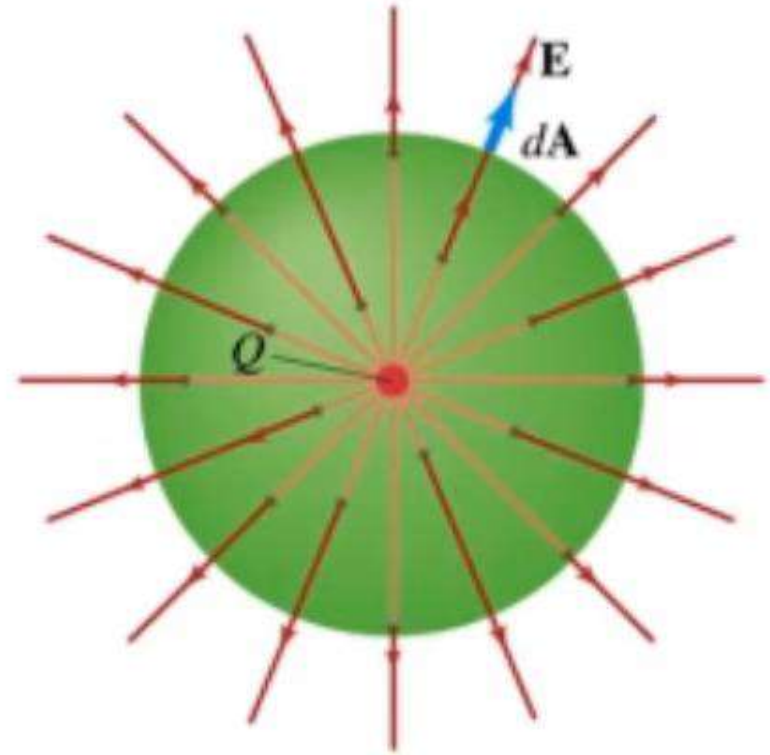
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$$E\oint dA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad EA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

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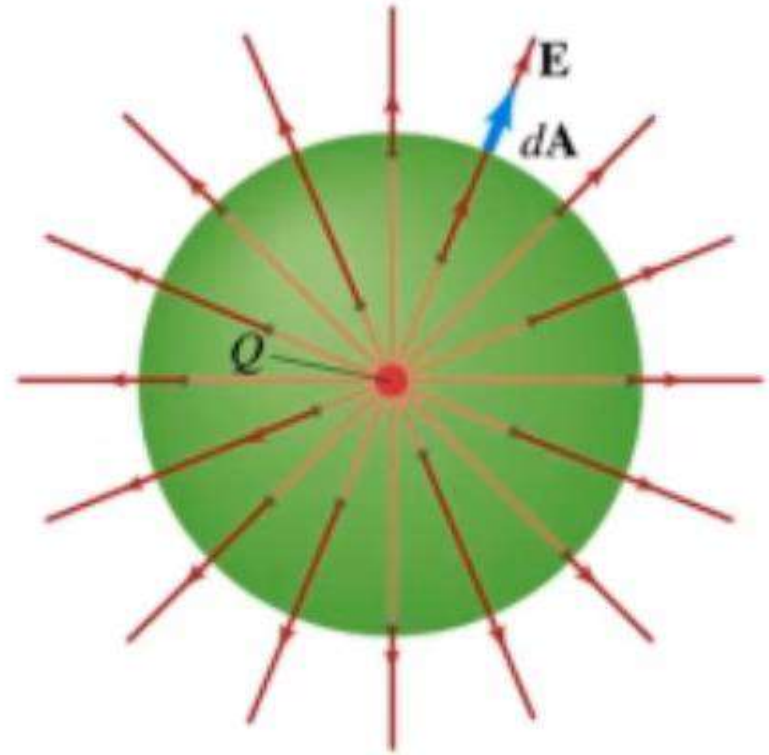
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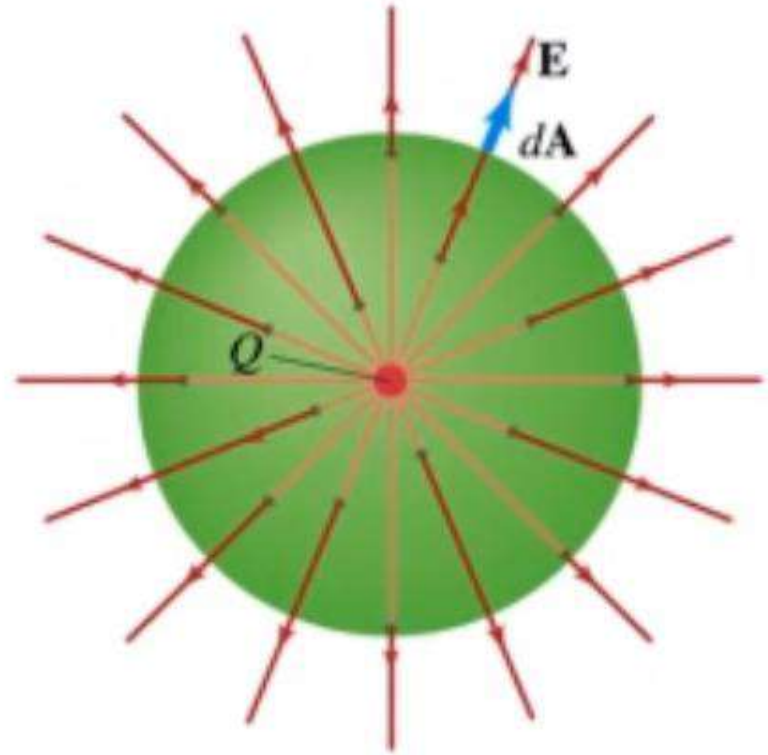
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$$E\oint dA = \frac{q_{in}}{\varepsilon_o} \Rightarrow EA = \frac{q_{in}}{\varepsilon_o} \Rightarrow E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o} \Rightarrow \therefore E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

Gauss's Law and Coulomb's Law

We can deduce Coulomb's law from Gauss's law by assuming a point charge *q*, to find the electric field at point or points a distance *r* from the charge we imagine a spherical Gaussian surface of radius *r* and the charge *q* at its center as shown in figure.



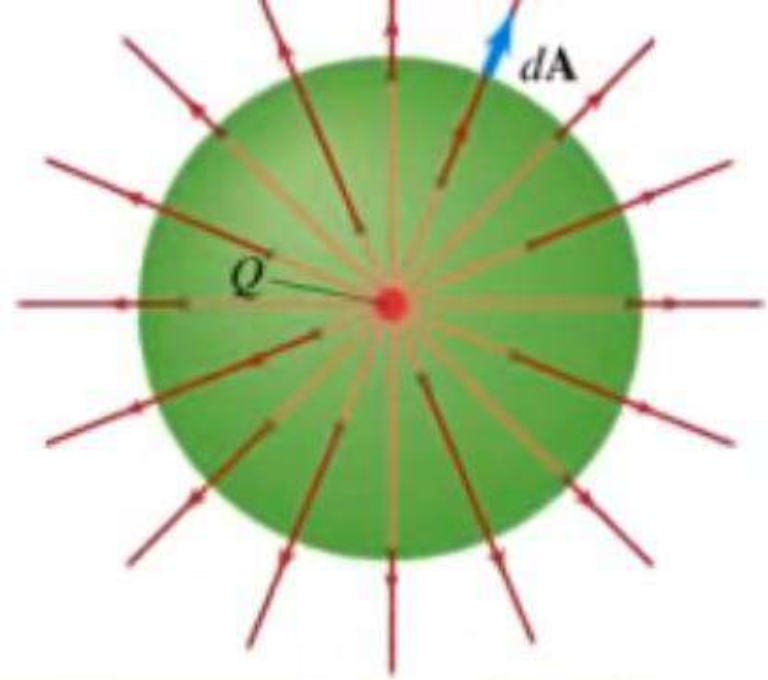
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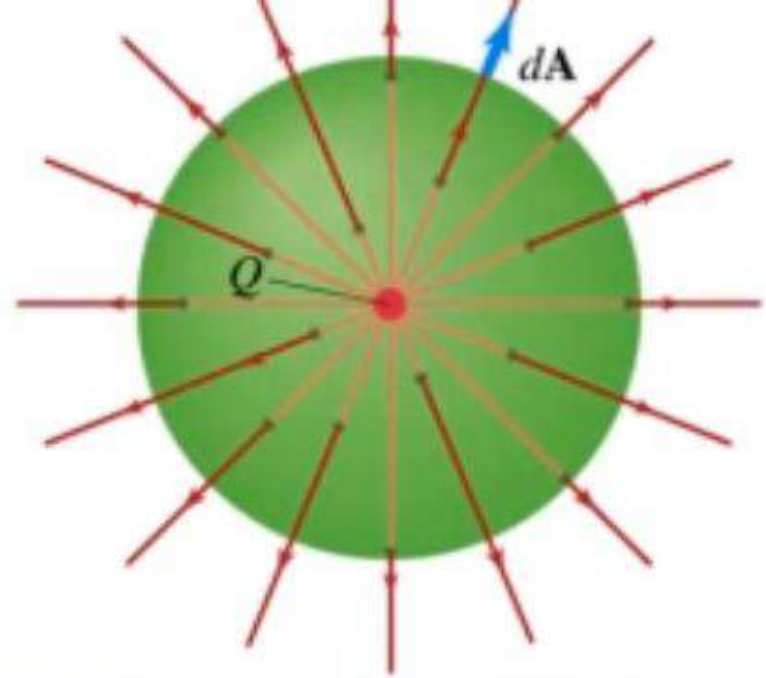
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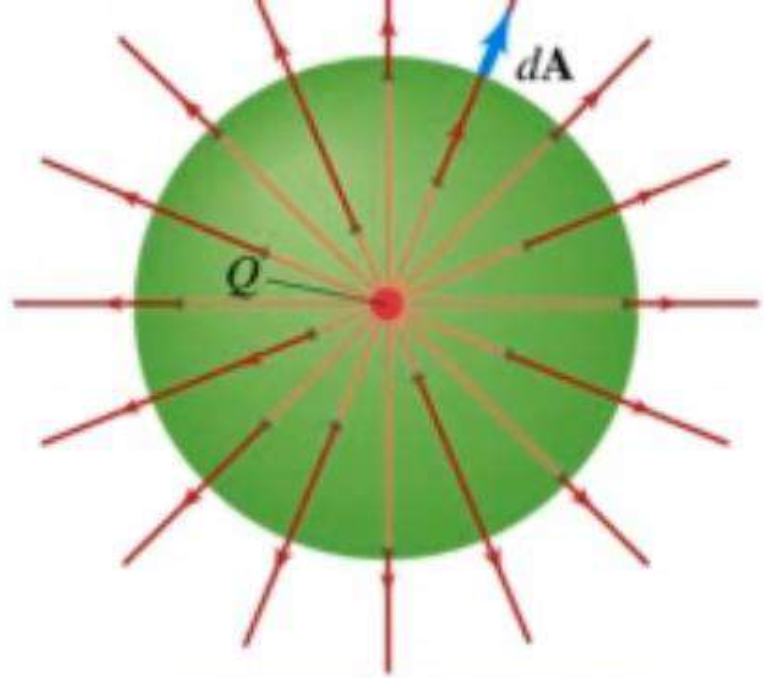
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$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

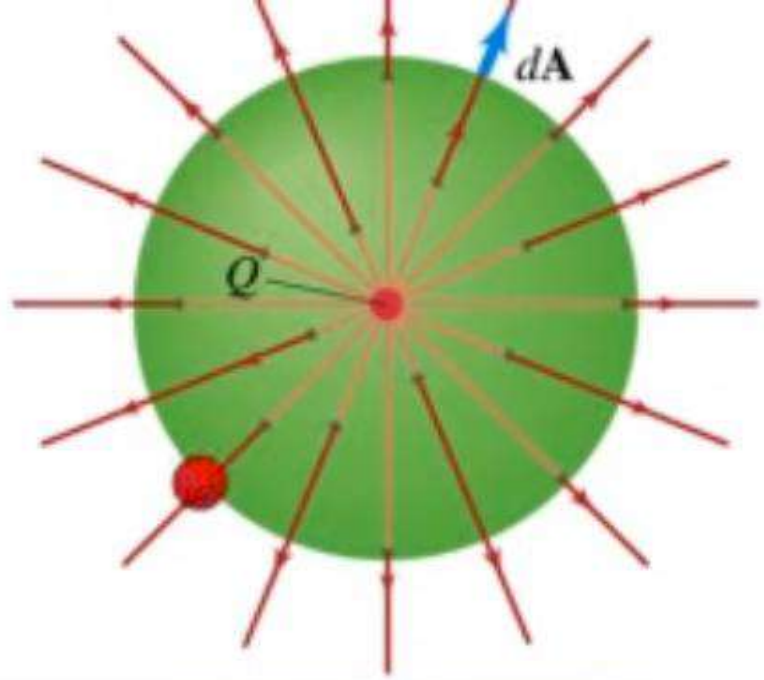
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Now put a second point charge q_o at the point, which E is calculated. The magnitude of the electric force that acts on it $F = Eq_o$

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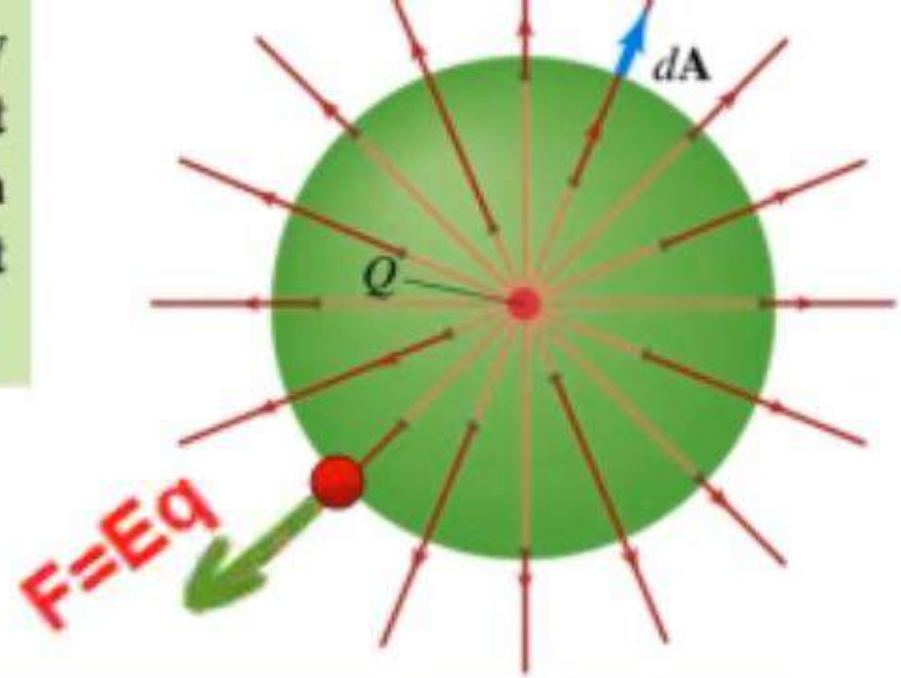
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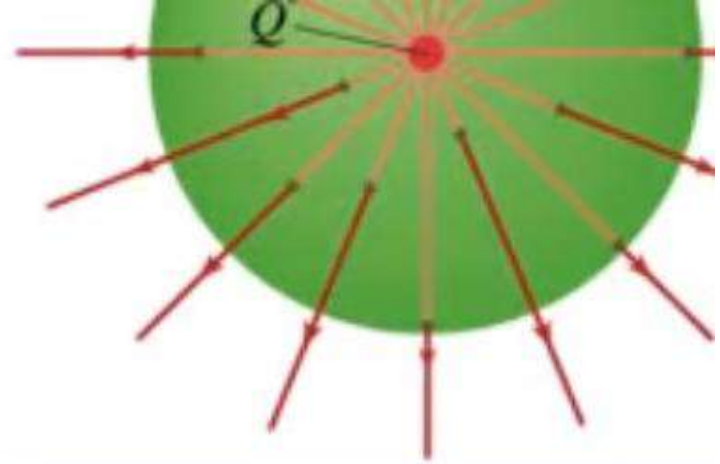
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Coulomb's law

$$\therefore F = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}$$

spherical Gaussian surface of radius r and the charge q at its center as shown in figure.



$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

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Now put a second point charge q_o at the point, which E is calculated. The magnitude of the electric force that acts on it $F = Eq_o$

Coulomb's law

$$\therefore F = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}$$

Application of Gauss's Law

- We want to compute the electric field at the surface of a charged metal object.
- This gives a good example of the application of Gauss's Law.
- First we establish some facts about good conductors.
- Then we can get a neat useful result:

$$E = \sigma / \varepsilon_0$$

Fields in Good Conductors

Fact: **In a steady state the electric field inside a good conductor must be zero.**

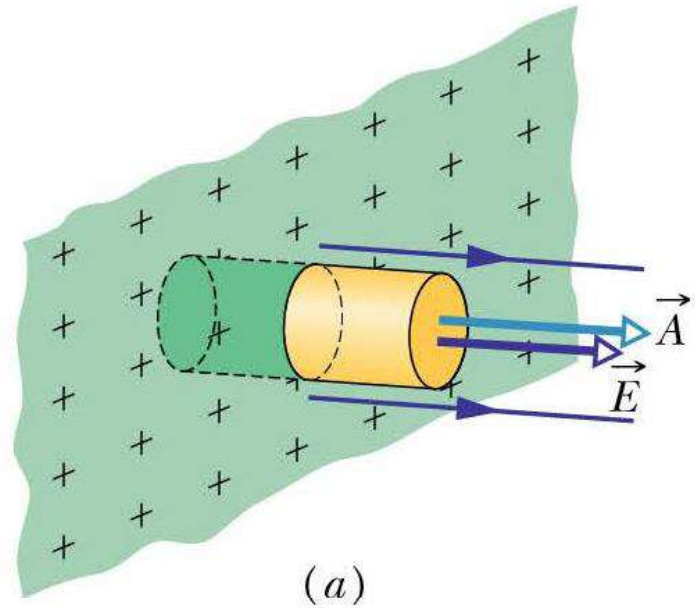
Why? **If there were a field, charges would move. Charges will move around until they find the arrangement that makes the electric field zero in the interior.**

Charges in Good Conductors

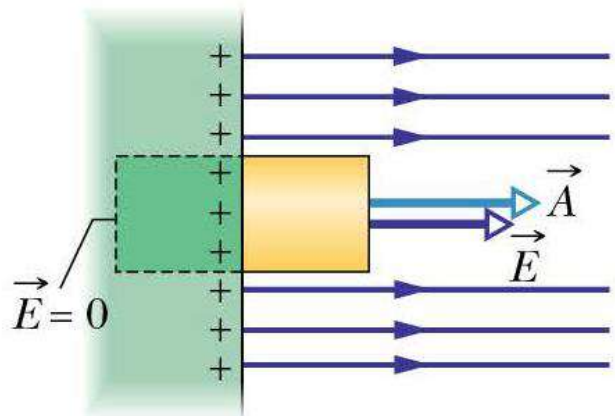
Fact: **In a steady state, any net charge on a good conductor must be entirely on the surface.**

Why? **If there were a charge in the interior, then by Gauss's Law there would be a field in the interior, which we know cannot be true.**

Field at the Surface of a Conductor



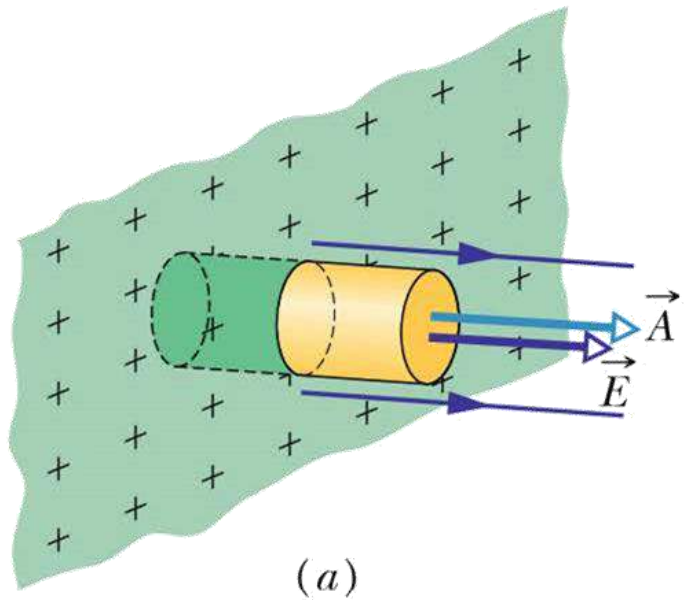
(a)



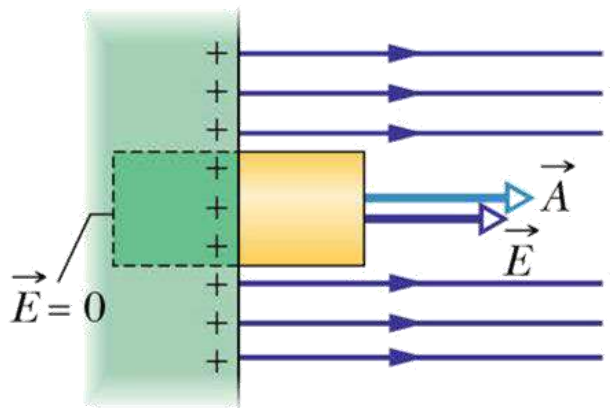
(b)

- Construct closed gaussian surface, sides perpendicular to metal surface, face area = A.
- Flux through left face is zero because E=0.
- Field is perpendicular to surface or charges would move, therefore flux through sides is 0.
- So net outward flux is EA.

Field at the Surface of a Conductor



(a)



(b)

- Let σ stand for the surface charge density (C/m^2)
- Then $Q_{enc} = \sigma A$.
- Now Gauss's Law gives us

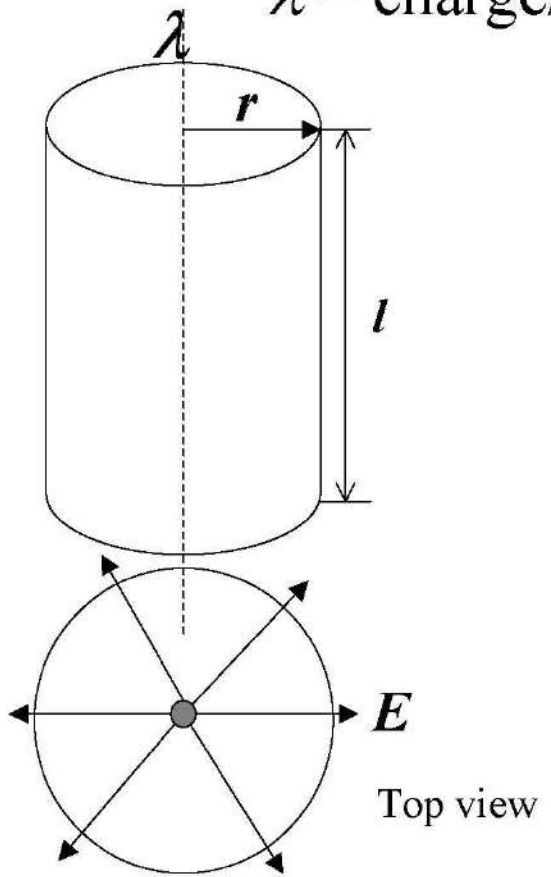
$$\Phi = Q_{enc} / \varepsilon_0$$

$$EA = \sigma A / \varepsilon_0$$

$$E = \sigma / \varepsilon_0$$

Long Line of Charge

λ = charge/length = linear charge density



$$\Phi = \frac{Q}{\varepsilon_0} \qquad \textbf{Gauss' law}$$

$$Q = l\lambda$$

$$\Phi = \oint \mathbf{E} \cdot d\mathbf{A} = EA$$

$$A = 2\pi r l$$

$$E = \frac{\lambda}{2\pi r \varepsilon_0}$$

Summary for different dimensions



Point charge, d=0

$$\Phi = 4\pi r^2 E = Q/\varepsilon_0$$

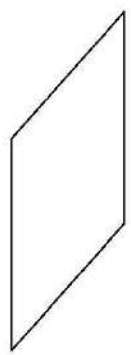
$$E = Q/(4\pi\varepsilon_0 r^2)$$

Line charge, d=1

$$E \propto \frac{1}{r^{2-d}}$$

$$\Phi = 2\pi rlE = \lambda l/\varepsilon_0$$

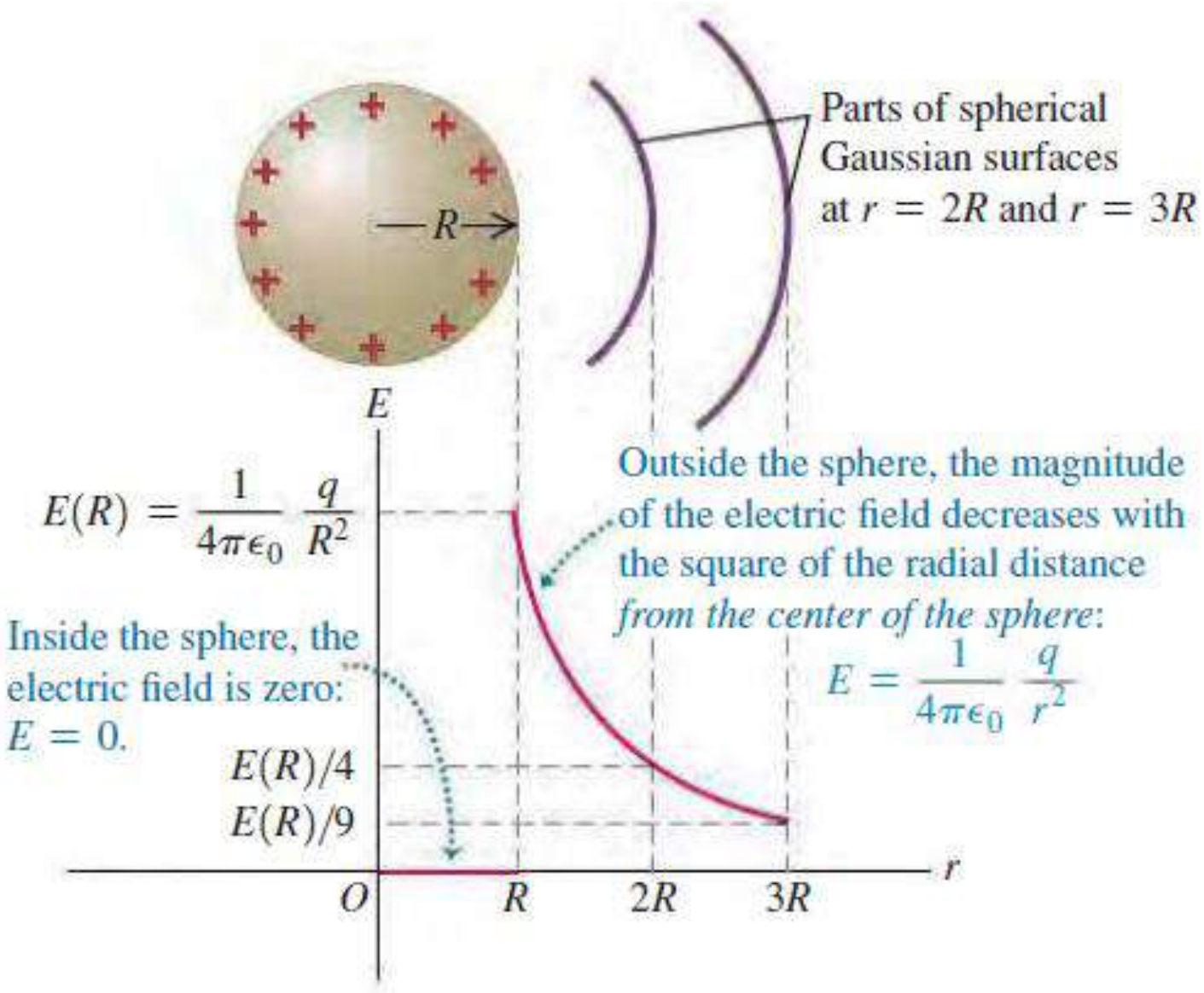
$$E = \lambda/(2\pi\varepsilon_0 r)$$



Surface charge, d=2

$$\Phi = 2AE = \sigma A/\varepsilon_0$$

$$E = \sigma/(2\varepsilon_0)$$



Calculating the electric field of a conducting sphere with positive charge q. Outside the sphere, the field is the same as if all of the charge were concentrated at the center of the sphere.

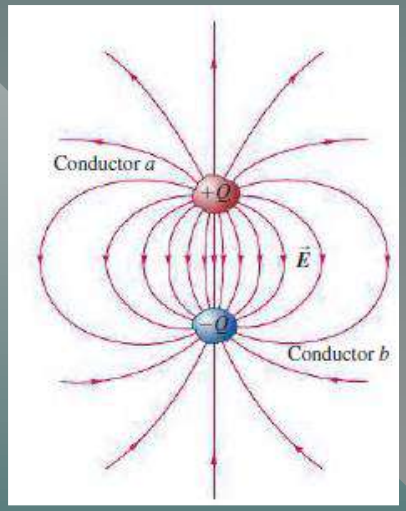

Radial Ceramic Capacitor
Three Terminal Capacitor
Wierd Ended Electrolytic Capacitor
Surface Mount Electrolytic
Motor Run Capacitor
Solid Chip Tanta
Surface Mount Ceramic Capacitor
Suppressor Capacitor
Polyster Capacitor
Polysterene Capacitor
Aluminium Electrolytic Capacitor
PCB Mount Electrolytic
Trimmer Capacitor
Polypropylyne Capacitor
Memory Back-up Capacitor
Capacitance
nichicon
4700μF 35v
189-505-5
102K
500V
151
561
10
224K
100J
15
22μF 35v

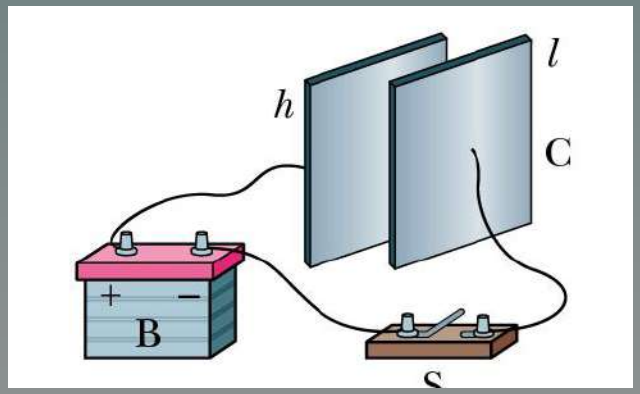
Capacitors

- Capacitors are devices that store electric charge
- Examples of where capacitors are used include:
  - radio receivers
  - filters in power supplies
  - to eliminate sparking in automobile ignition systems
  - energy-storing devices in electronic flashes

Capacitors

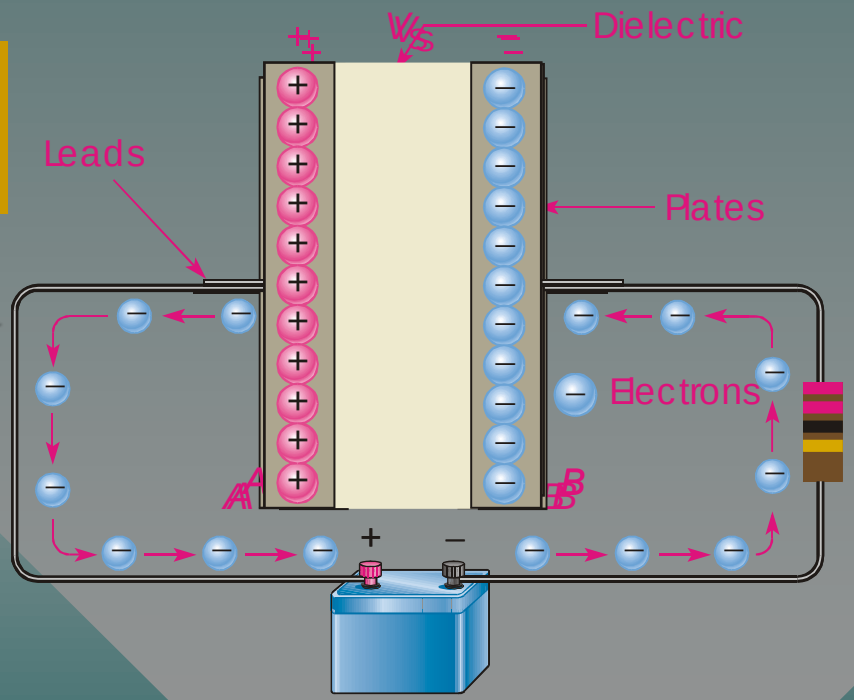
Consider two separated conductors, like two parallel plates, with external leads to attach to other circuit elements. Such a device is called a capacitor.





There is a limit to the amount of charge that a conductor can hold without leaking to the air. There is a certain capacity for holding charge.

The charging process…



A capacitor with stored charge can act as a temporary battery.

Definition of Capacitance

- The **capacitance**, *C*, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the potential difference between the conductors



- The SI unit of capacitance is the Farad (F)

Capacitance in Farads

One farad (F) is the capacitance C of a conductor that holds one coulomb of charge for each volt of potential.

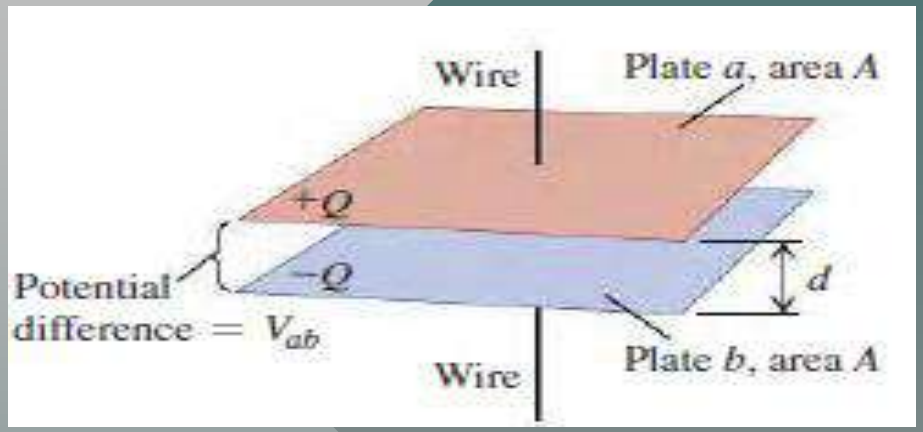
$$C = \frac{Q}{V}; \quad farad\ (\mathrm{F}) = \frac{coulomb\ (\mathrm{C})}{volt\ (\mathrm{V})}$$

Example: When 40 μC of charge are placed on a conductor, the potential is 8 V. What is the capacitance?

$$C = \frac{Q}{V} = \frac{40\ \mu\mathrm{C}}{8\ \mathrm{V}}$$

C = 5 μF

Parallel Plate Capacitance



For these two parallel plates:

$$C = \frac{Q}{V} \quad \text{and} \quad E = \frac{V}{d}$$

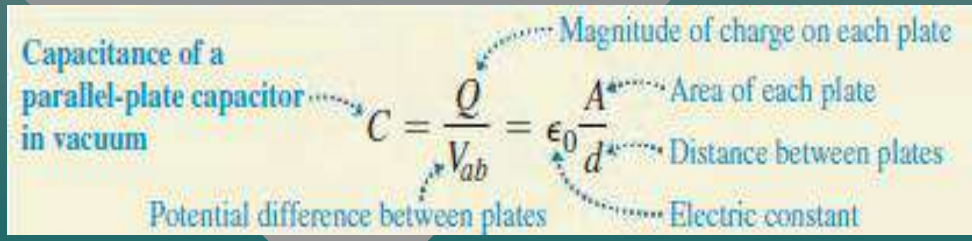
You will recall from Gauss' law that E is also:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

Q is charge on either plate. *A* is area of plate.

$$E = \frac{V}{d} = \frac{Q}{\varepsilon_0 A}$$

And



EXAMPLE 24.2 PROPERTIES OF A PARALLEL-PLATE CAPACITOR

SOLUTION

The plates of a parallel-plate capacitor in vacuum are 5.00 mm apart and 2.00 m^2 in area. A 10.0-kV potential difference is applied across the capacitor. Compute (a) the capacitance; (b) the charge on each plate; and (c) the magnitude of the electric field between the plates.

SOLUTION

IDENTIFY and SET UP: We are given the plate area A, the plate spacing d, and the potential difference $V_{ab} = 1.00 \times 10^4$ V for this parallel-plate capacitor. Our target variables are the capacitance C, the charge Q on each plate, and the electric-field magnitude E. We use Eq. (24.2) to calculate C and then use Eq. (24.1) and V_{ab} to find Q. We use $E = Q/\epsilon_0 A$ to find E.

EXECUTE: (a) From Eq. (24.2),

$$C = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}\ \text{F/m}) \frac{(2.00\ \text{m}^2)}{5.00 \times 10^{-3}\ \text{m}}$$
$$= 3.54 \times 10^{-9}\ \text{F} = 0.00354\ \mu\text{F}$$

(b) The charge on the capacitor is

$$Q = CV_{ab} = (3.54 \times 10^{-9}\ \text{C/V})(1.00 \times 10^4\ \text{V})$$
$$= 3.54 \times 10^{-5}\ \text{C} = 35.4\ \mu\text{C}$$

The plate at higher potential has charge $+35.4\ \mu$C, and the other plate has charge $-35.4\ \mu$C.

(c) The electric-field magnitude is

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} = \frac{3.54 \times 10^{-5}\ \text{C}}{(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2)(2.00\ \text{m}^2)}$$
$$= 2.00 \times 10^6\ \text{N/C}$$

EVALUATE: We can also find E by recalling that the electric field is equal in magnitude to the potential gradient [Eq. (23.22)]. The field between the plates is uniform, so

$$E = \frac{V_{ab}}{d} = \frac{1.00 \times 10^4\ \text{V}}{5.00 \times 10^{-3}\ \text{m}} = 2.00 \times 10^6\ \text{V/m}$$

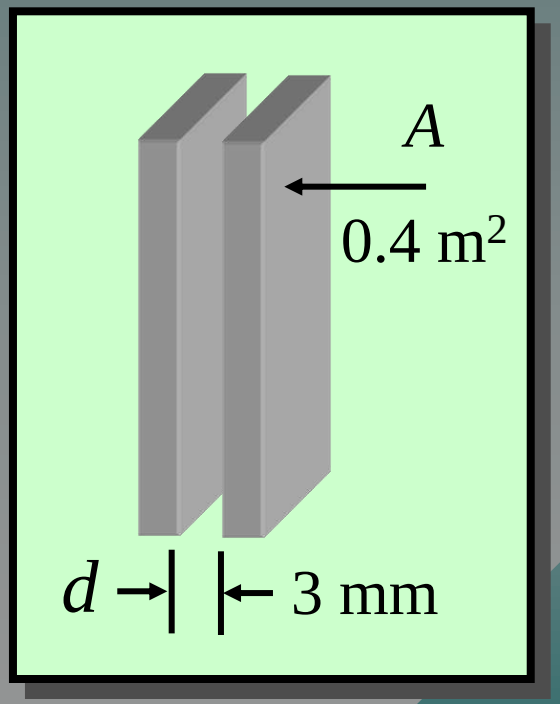
(Remember that 1 N/C = 1 V/m.)

Example : The plates of a parallel plate capacitor have an area of 0.4 m^2 and are 3 mm apart in air. What is the capacitance?

$$C = \frac{Q}{V} = \varepsilon_0 \frac{A}{d}$$

$$C = \frac{(8.85 \text{ x } 10^{-12} \frac{C^2}{Nm^2})(0.4 \text{ m}^2)}{(0.003 \text{ m})}$$

C = 1.18 nF



Energy of Charged Capacitor

The potential energy U of a charged capacitor is equal to the work (qV) required to charge the capacitor.

If we consider the average potential difference from 0 to V_f to be V/2:

Work = Q(V/2) = ½QV

$$U = \tfrac{1}{2}QV; \quad U = \tfrac{1}{2}CV^2; \quad U = \frac{Q^2}{2C}$$

Example: In a capacitor, we found its capacitance to be 11.1 nF, the voltage 200 V, and the charge 2.22 μC. Find the potential energy U.

$$U = \tfrac{1}{2}CV^2$$

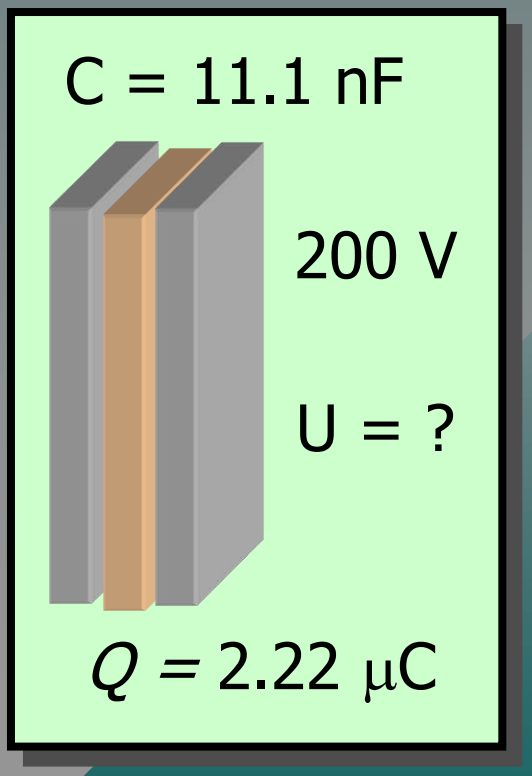
$$U = \tfrac{1}{2}(11.1\text{ nF})(200\text{ V})^2$$

$$U = 222\ \mu\text{J}$$

Verify your answer from the other formulas for P.E.

$$U = \tfrac{1}{2}QV; \quad U = \frac{Q^2}{2C}$$

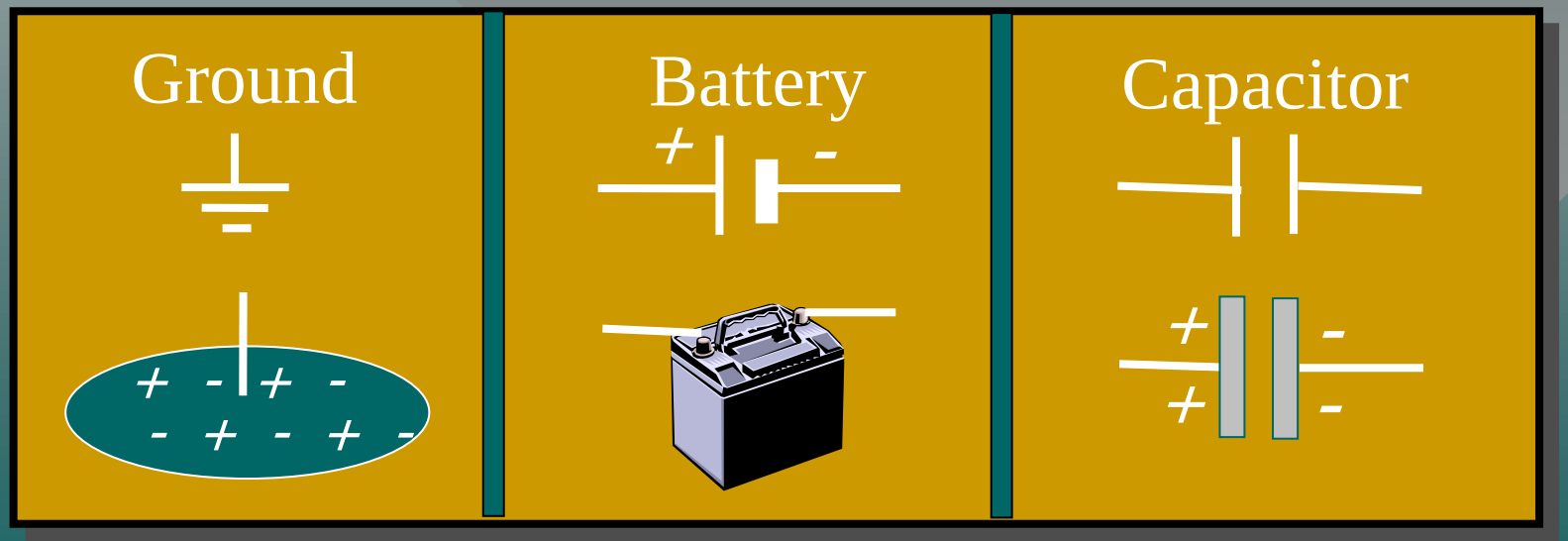
Capacitor of Example 3.



Electrical Circuit Symbols

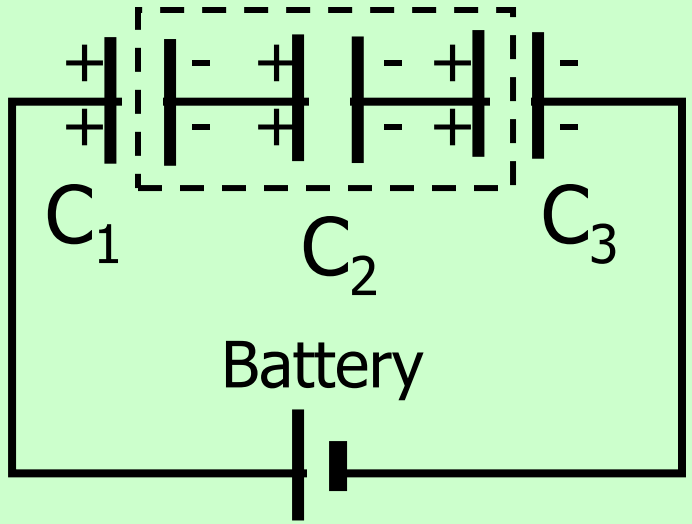
Electrical circuits often contain two or more capacitors grouped together and attached to an energy source, such as a battery.

The following symbols are often used:



Capacitors in Series

Capacitors or other devices connected along a single path are said to be connected in series. See circuit below:

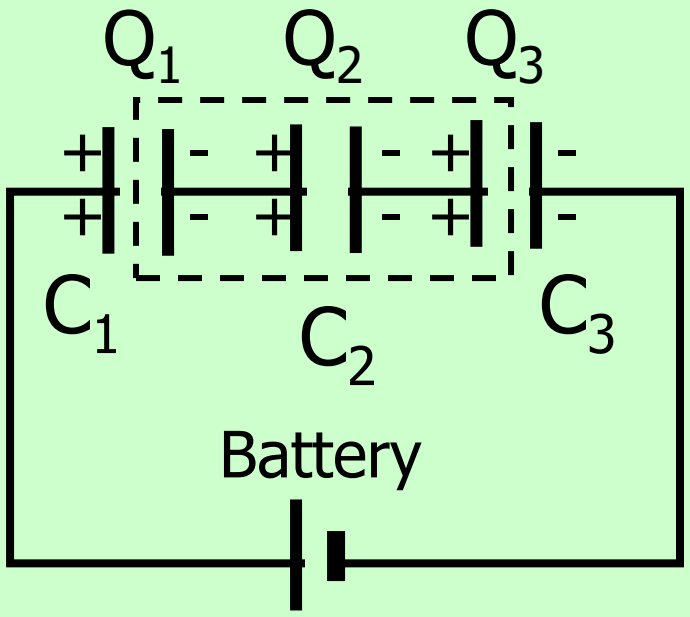


Series connection of capacitors. "+ to – to + …"

Charge inside dots is induced.

Charge on Capacitors in Series

Since inside charge is only induced, the charge on each capacitor is the same.

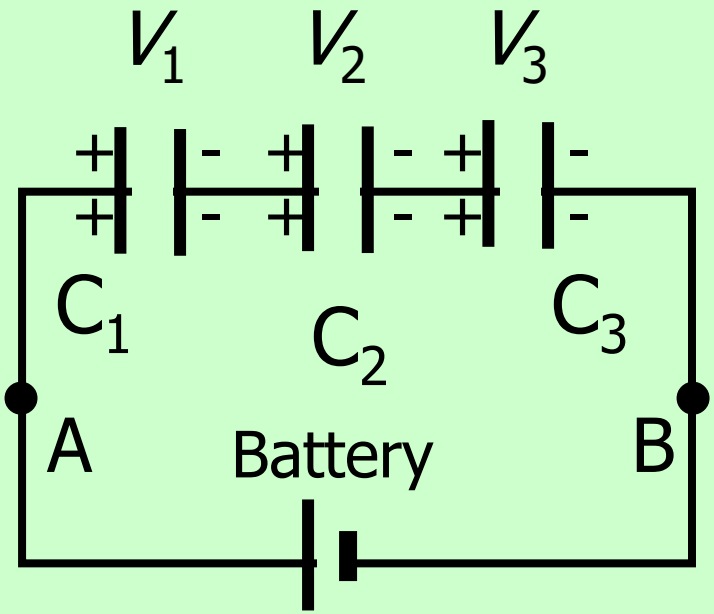


Charge is same: series connection of capacitors.

$$Q = Q_1 = Q_2 = Q_3$$

Voltage on Capacitors in Series

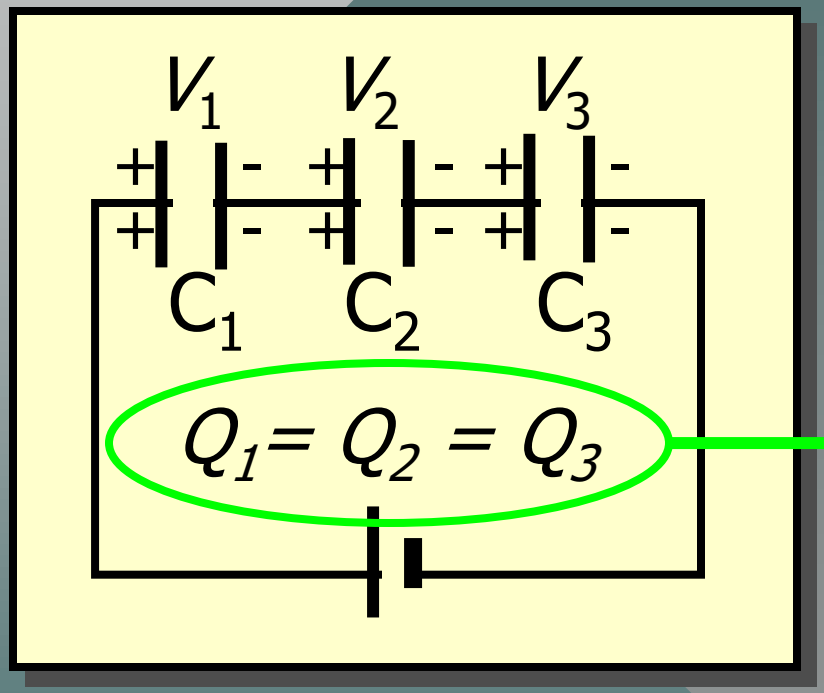
Since the potential difference between points A and B is independent of path, the battery voltage V must equal the sum of the voltages across each capacitor.



Total voltage V
Series connection
Sum of voltages

$$V = V_1 + V_2 + V_3$$

Equivalent Capacitance: Series



$$C = \frac{Q}{V}; \quad V = \frac{Q}{C}$$

$$V = V_1 + V_2 + V_3$$

$$\frac{\cancel{Q}}{C} = \frac{\cancel{Q_1}}{C_1} + \frac{\cancel{Q_2}}{C_2} + \frac{\cancel{Q_3}}{C_3}$$

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

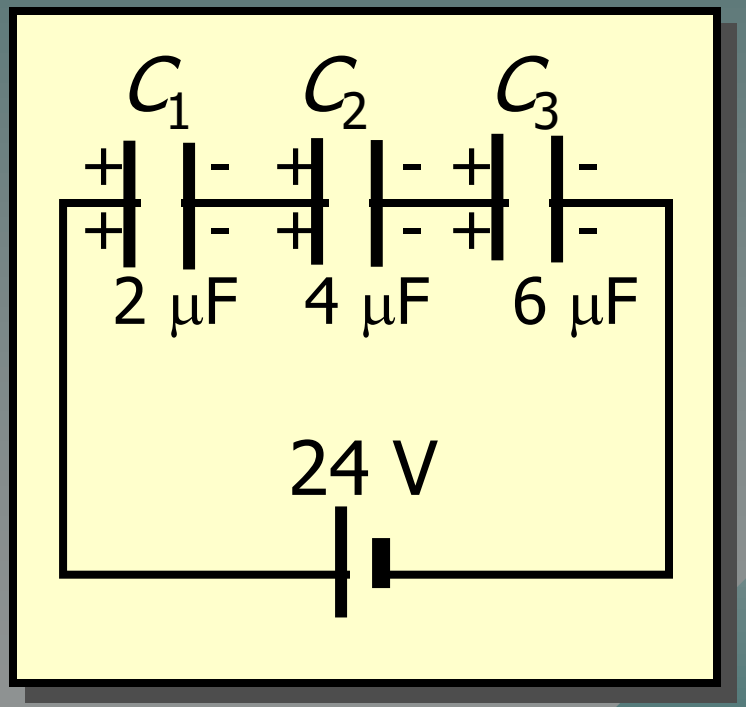
Equivalent C_e for capacitors in series:

$$\frac{1}{C_e} = \sum_{i=1}^{n} \frac{1}{C_i}$$

Example Find the equivalent capacitance of the three capacitors connected in series with a 24-V battery.

C_e for series:

$$\frac{1}{C_e} = \sum_{i=1}^{n} \frac{1}{C_i}$$



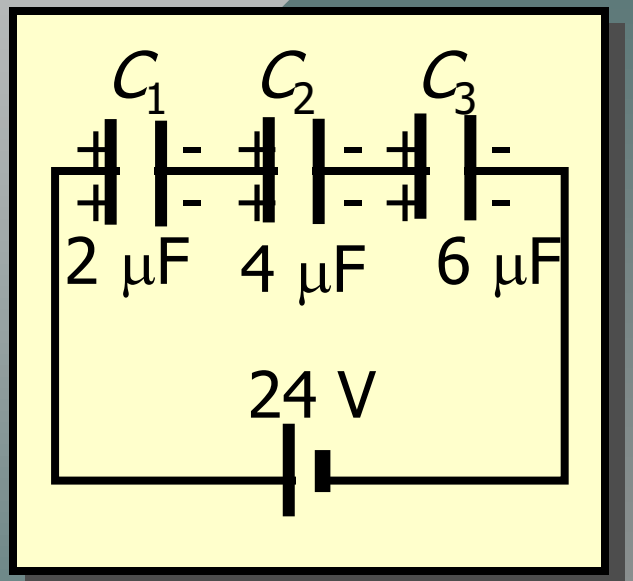
$$\frac{1}{C_e} = \frac{1}{2\mu F} + \frac{1}{4\mu F} + \frac{1}{6\mu F}$$

$$\frac{1}{C_e} = 0.500 + 0.250 + 0.167$$

$$\frac{1}{C_e} = 0.917 \quad \text{or} \quad C_e = \frac{1}{0.917}$$

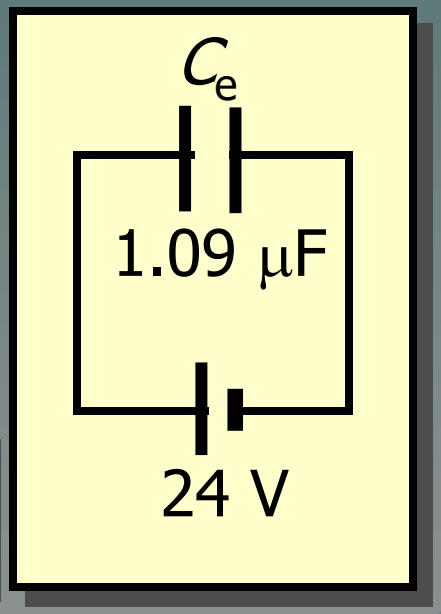
$$C_e = 1.09\ \mu F$$

Example (Cont.): The equivalent circuit can be shown as follows with single C_e.



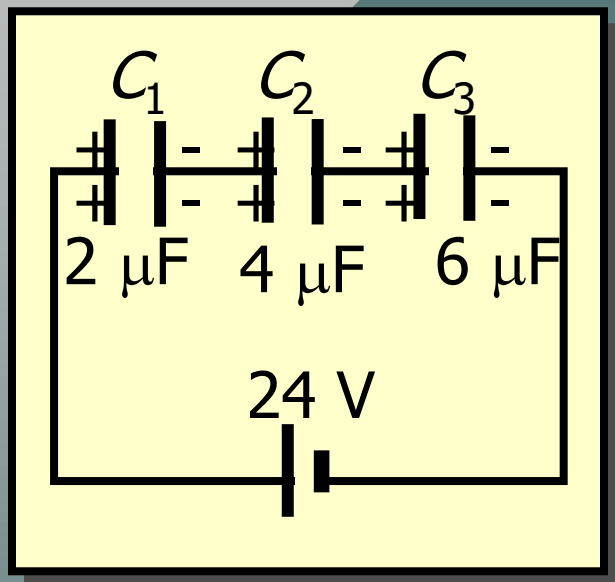
$$\frac{1}{C_e} = \sum_{i=1}^{n} \frac{1}{C_i}$$

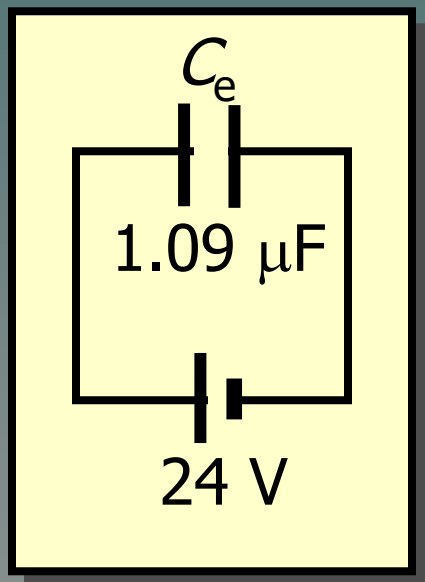
$$C_e = 1.09\ \mu F$$



Note that the equivalent capacitance C_e for capacitors in series is always less than the least in the circuit. (1.09 μF < 2 μF)

Example (Cont.): What is the total charge and the charge on each capacitor?





$C_e = 1.09\ \mu F$

$$C = \frac{Q}{V}$$

$$Q = CV$$

$Q_T = C_e V = (1.09\ \mu F)(24\ V);$ $\quad Q_T = 26.2\ \mu C$

For series circuits:

$Q_T = Q_1 = Q_2 = Q_3$

$Q_1 = Q_2 = Q_3 = 26.2\ \mu C$

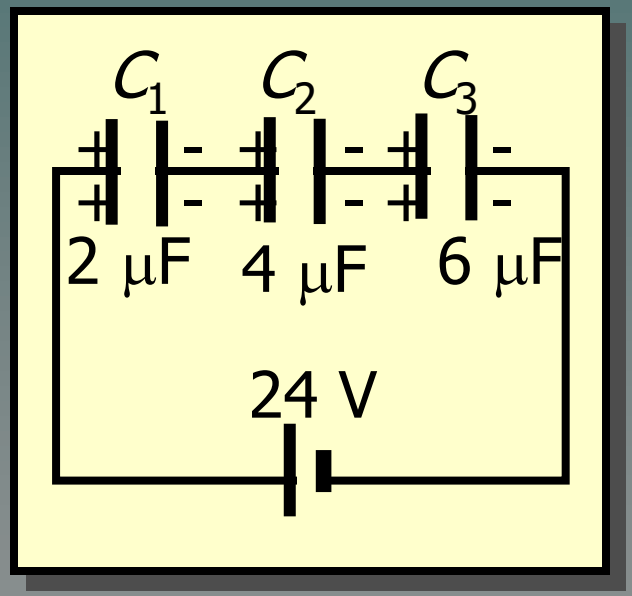
Example (Cont.): What is the voltage across each capacitor?

$$C = \frac{Q}{V}; \quad V = \frac{Q}{C}$$

$$V_1 = \frac{Q_1}{C_1} = \frac{26.2\ \mu\text{C}}{2\ \mu\text{F}} = 13.1\ \text{V}$$

$$V_2 = \frac{Q_2}{C_2} = \frac{26.2\ \mu\text{C}}{4\ \mu\text{F}} = 6.55\ \text{V}$$

$$V_3 = \frac{Q_3}{C_3} = \frac{26.2\ \mu\text{C}}{6\ \mu\text{F}} = 4.37\ \text{V}$$



$V_T = 24\ \text{V}$

Note: $V_T = 13.1\ \text{V} + 6.55\ \text{V} + 4.37\ \text{V} = 24.0\ \text{V}$

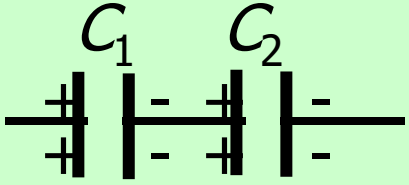
Short Cut: Two Series Capacitors

The equivalent capacitance C_e for two series capacitors is the product divided by the sum.

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2};$$

$$C_e = \frac{C_1 C_2}{C_1 + C_2}$$

Example:



$$C_e = \frac{(3\ \mu\text{F})(6\ \mu\text{F})}{3\ \mu\text{F} + 6\mu\text{F}}$$

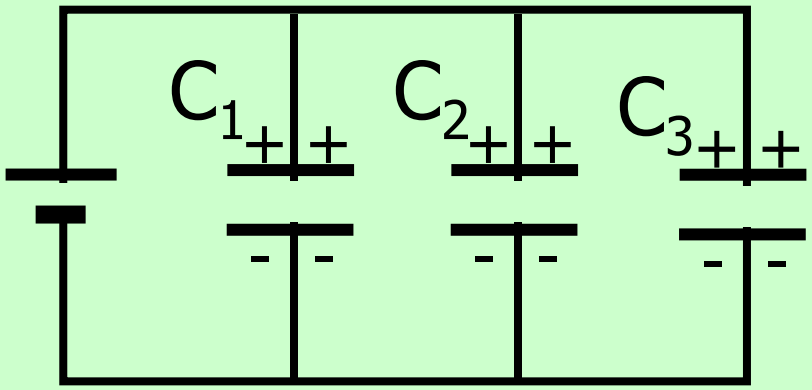
$$C_e = 2\ \mu\text{F}$$

Parallel Circuits

Capacitors which are all connected to the same source of potential are said to be connected in parallel. See below:

Parallel capacitors:

"+ to +; - to -"



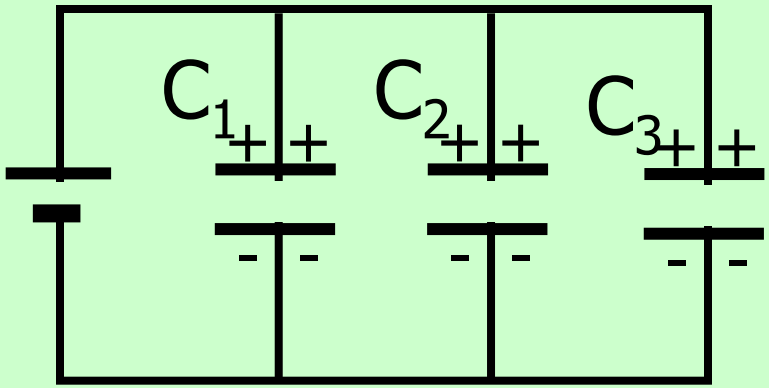
Voltages:

$V_T = V_1 = V_2 = V_3$

Charges:

$Q_T = Q_1 + Q_2 + Q_3$

Parallel capacitors in Parallel:



$$C = \frac{Q}{V}; \quad Q = CV$$

$$Q = Q_1 + Q_2 + Q_3$$

Equal Voltages:

$$CV = C_1V_1 + C_2V_2 + C_3V_3$$

$$C_e = C_1 + C_2 + C_3$$

Equivalent C_e for capacitors in parallel: $C_e = \sum_{i=1}^{n} C_i$

Example Find the equivalent capacitance of the three capacitors connected in parallel with a 24-V battery.

C_e for parallel:

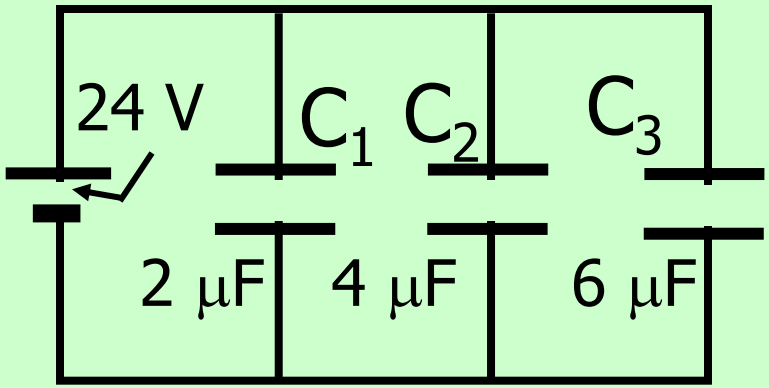
$$C_e = \sum_{i=1}^{n} C_i$$

$$C_e = (2 + 4 + 6)\ \mu F$$

$$C_e = 12\ \mu F$$

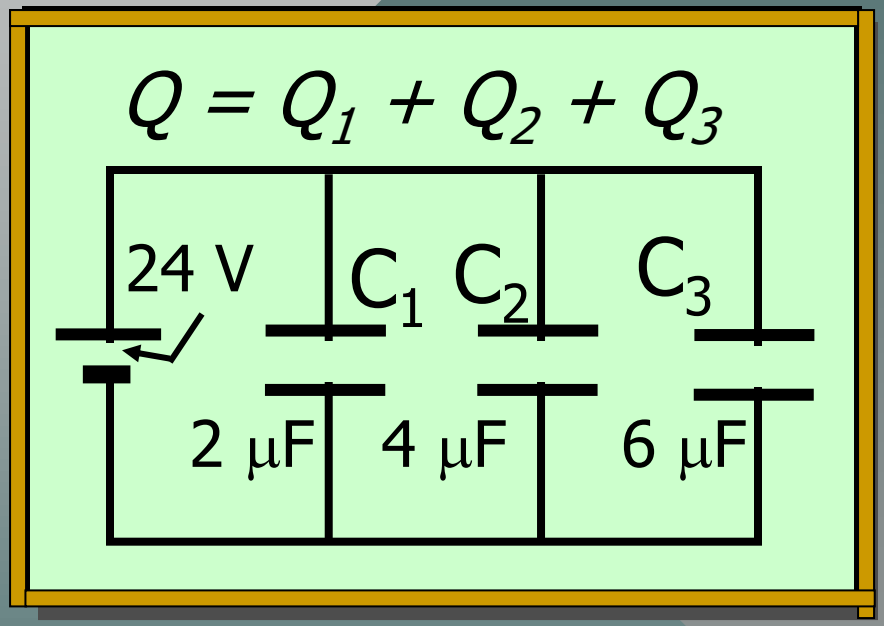
$$V_T = V_1 = V_2 = V_3$$

$$Q = Q_1 + Q_2 + Q_3$$



Note that the equivalent capacitance C_e for capacitors in parallel is always greater than the largest in the circuit. (12 μF > 6 μF)

Example (Cont.) Find the total charge Q_T and charge across each capacitor.



$$C_e = 12\ \mu F$$

$$V_1 = V_2 = V_3 = 24\ V$$

$$C = \frac{Q}{V}; \quad Q = CV$$

$$Q_T = C_e V$$

$$Q_T = (12\ \mu F)(24\ V)$$

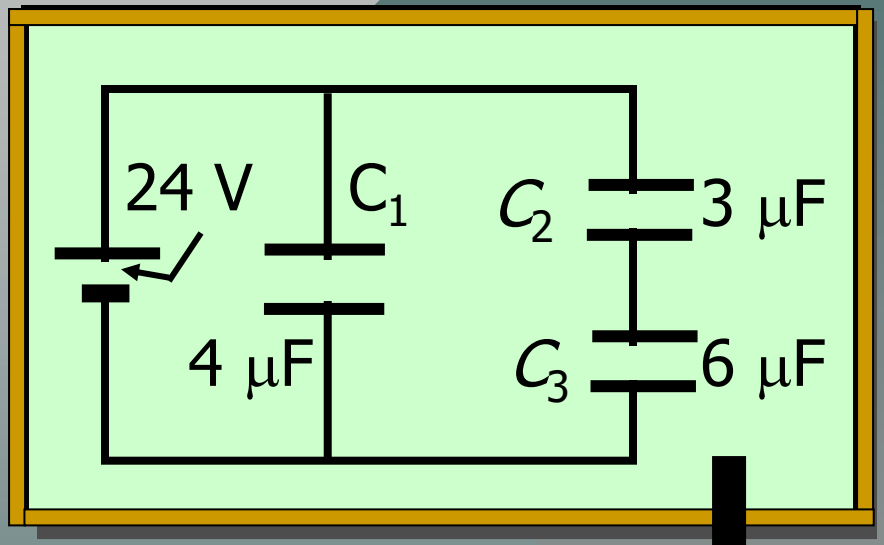
$$Q_T = 288\ \mu C$$

$$Q_1 = (2\ \mu F)(24\ V) = 48\ \mu C$$

$$Q_1 = (4\ \mu F)(24\ V) = 96\ \mu C$$

$$Q_1 = (6\ \mu F)(24\ V) = 144\ \mu C$$

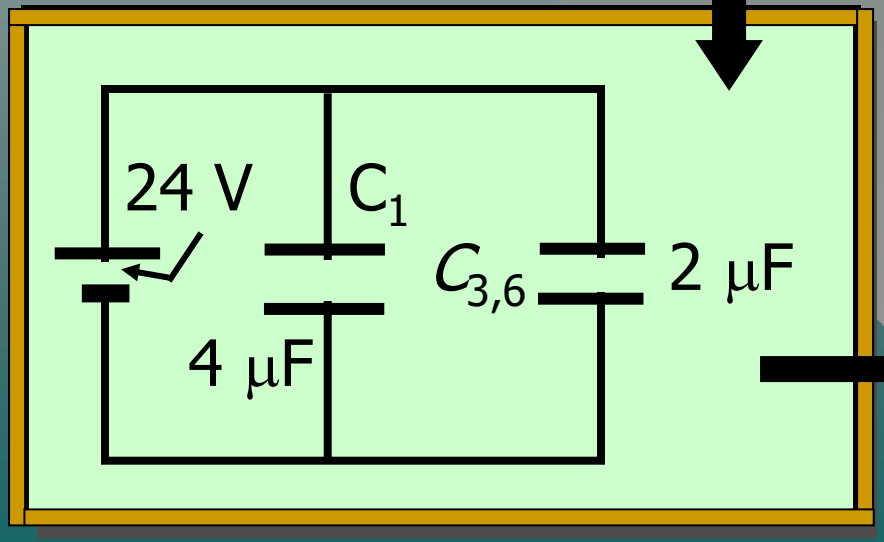
Example Find the equivalent capacitance of the circuit drawn below.

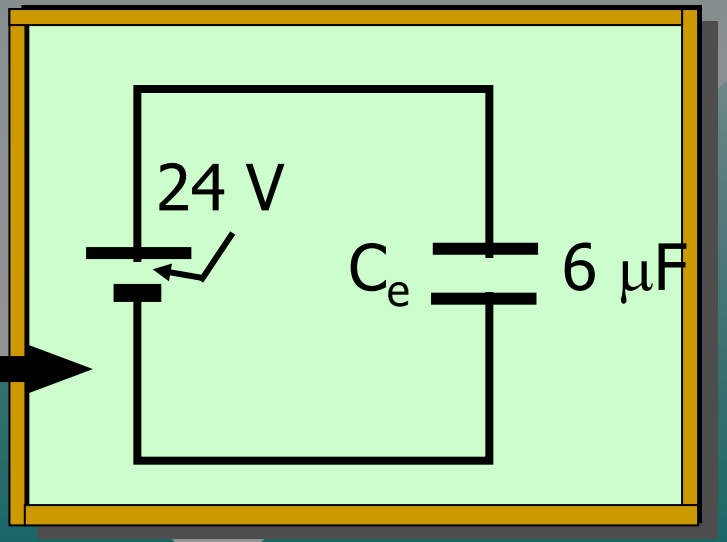


$$C_{3,6} = \frac{(3\mu\text{F})(6\mu\text{F})}{3\mu\text{F} + 6\mu\text{F}} = 2\mu\text{F}$$

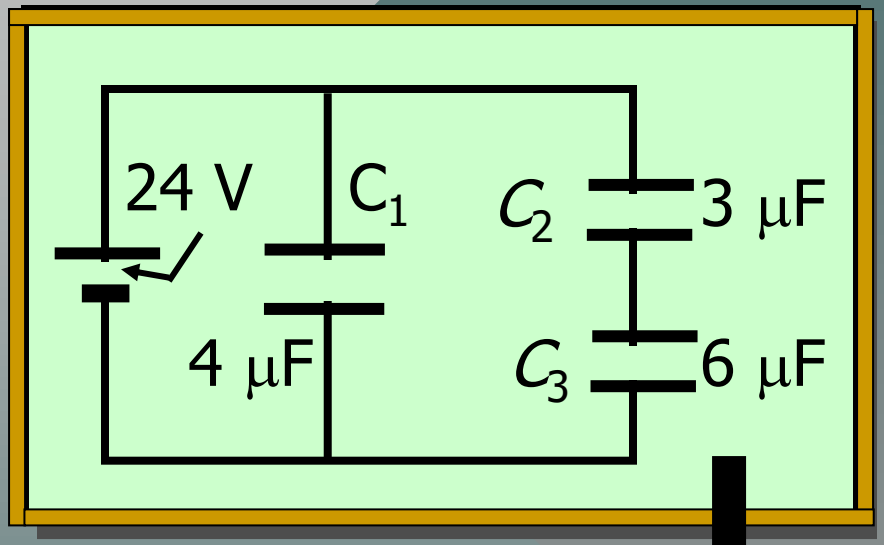
$C_e = 4\ \mu\text{F} + 2\ \mu\text{F}$

$C_e = 6\ \mu\text{F}$





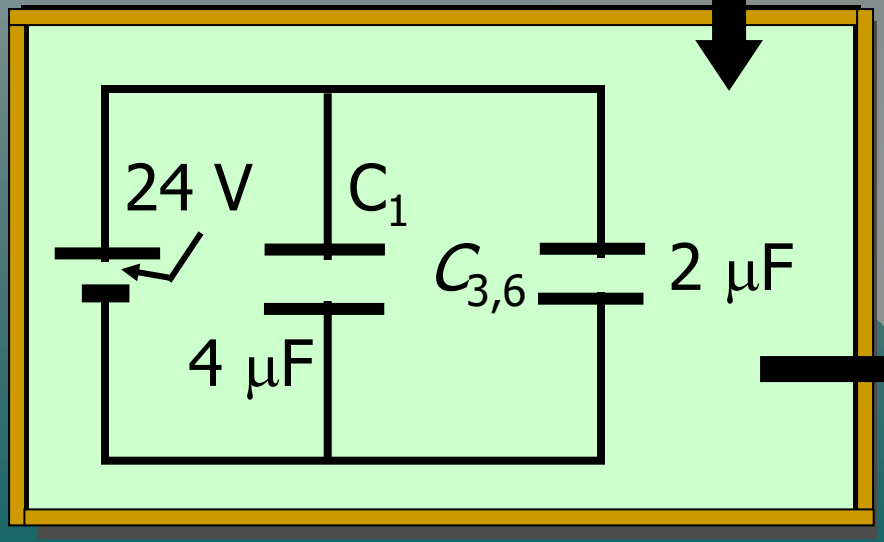
Example (Cont.) Find the total charge Q_T.

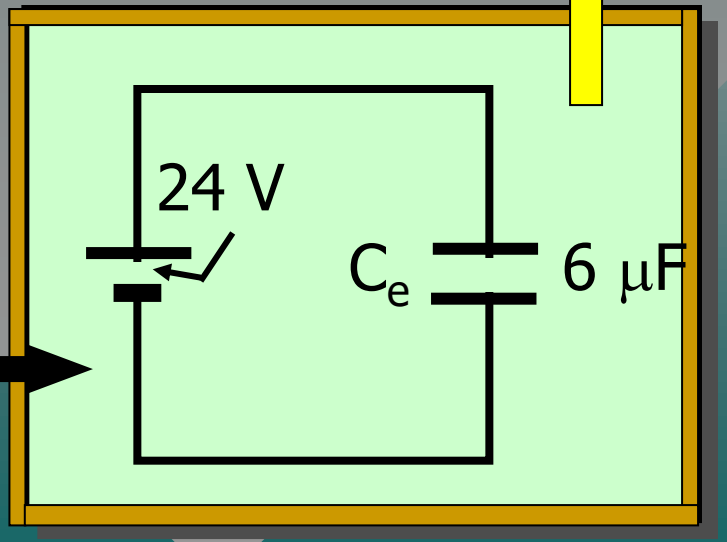


$$C_e = 6\ \mu F$$

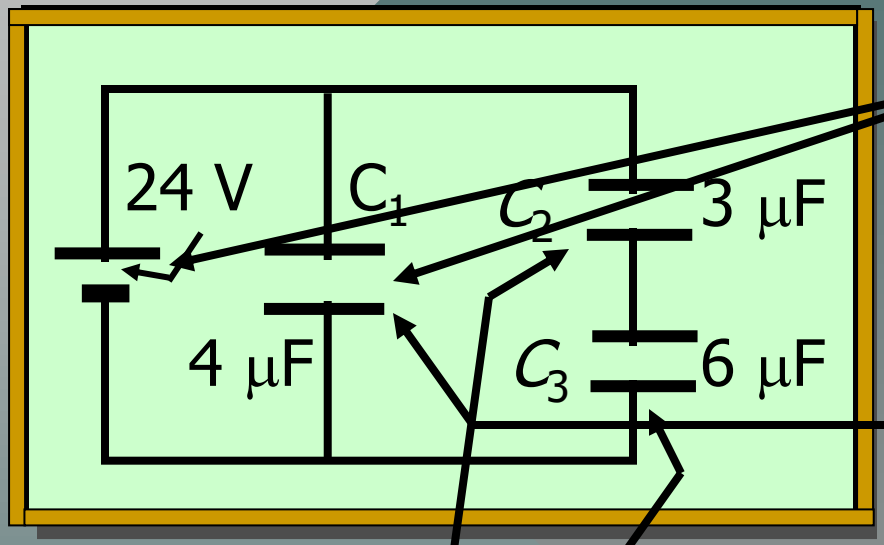
$$Q = CV = (6\ \mu F)(24\ V)$$

$$Q_T = 144\ \mu C$$





Example (Cont.) Find the charge Q_4 and voltage V_4 across the the 4–μF capacitor.



$V_4 = V_T = 24\,\text{V}$

$Q_4 = (4\ \mu\text{F})(24\ \text{V})$

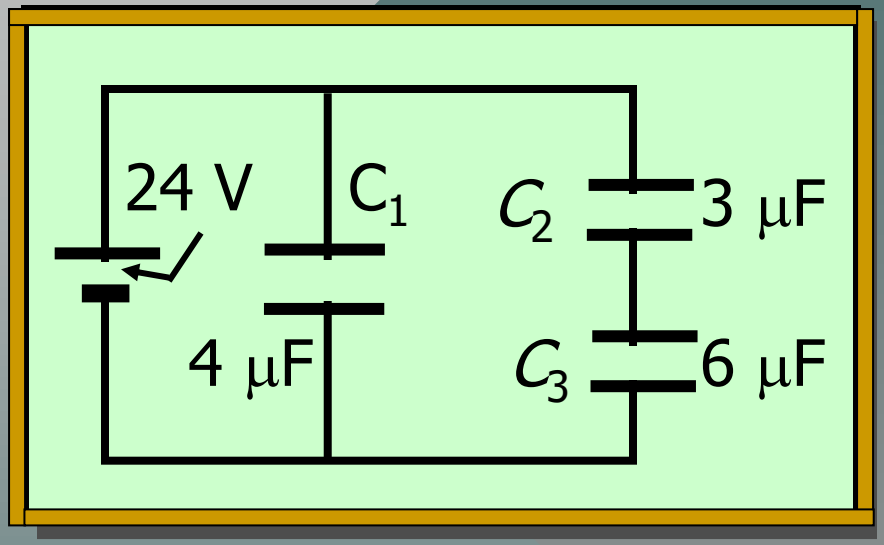
$Q_4 = 96\ \mu\text{C}$

The remainder of the charge: (144 μC – 96 μC) is on EACH of the other capacitors. (Series)

$Q_3 = Q_6 = 48\ \mu\text{C}$

This can also be found from $Q = C_{3,6}V_{3,6} = (2\ \mu\text{F})(24\ \text{V})$

Example (Cont.) Find the voltages across the 3 and 6-μF capacitors.



$$Q_3 = Q_6 = 48\ \mu\text{C}$$

$$V_3 = \frac{48\mu\text{C}}{3\mu\text{F}} = 16.0\text{V}$$

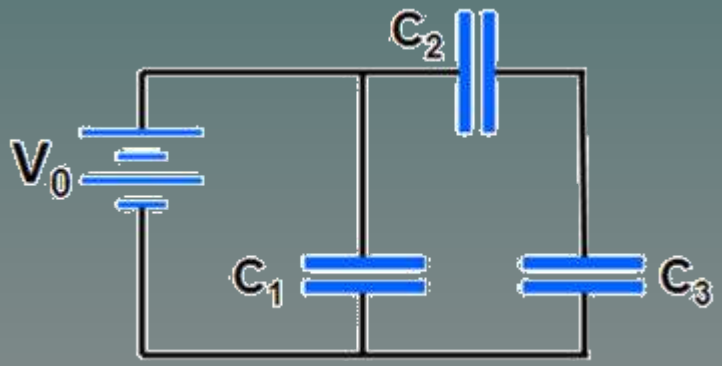
$$V_6 = \frac{48\mu\text{C}}{6\mu\text{F}} = 8.00\text{V}$$

Note: $V_3 + V_6 = 16.0\ \text{V} + 8.00\ \text{V} = 24\ \text{V}$

Use these techniques to find voltage and capacitance across each capacitor in a circuit.

H.W

A circuit consists of three unequal capacitors C_1, C_2, and C_3 which are connected to a battery of voltage V_0. The capacitance of C_2 is twice that of C_1. The capacitance of C_3 is three times that of C_1. The capacitors obtain charges Q_1, Q_2, and Q_3.



Compare Q_1, Q_2, and Q_3.

A. $Q_1 > Q_3 > Q_2$ ***B.*** $Q_1 > Q_2 > Q_3$ ***C.*** $Q_1 > Q_2 = Q_3$ ***D.*** $Q_1 = Q_2 = Q_3$ ***E.*** $Q_1 < Q_2 = Q_3$

Summary: Series Circuits

$$\frac{1}{C_e} = \sum_{i=1}^{n} \frac{1}{C_i}$$

$$Q = Q_1 = Q_2 = Q_3$$

$$V = V_1 + V_2 + V_3$$

For two capacitors at a time:

$$C_e = \frac{C_1 C_2}{C_1 + C_2}$$

Summary: Parallel Circuits

$$C_e = \sum_{i=1}^{n} C_i$$

$$Q = Q_1 + Q_2 + Q_3$$

$$V = V_1 = V_2 = V_3$$

For complex circuits, reduce the circuit in steps using the rules for both series and parallel connections until you are able to solve problem.

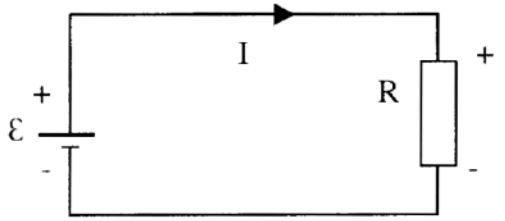
Current, Resistance, and Electromotive Force

Current

Current is the motion of any charge, positive or negative, from one point to another

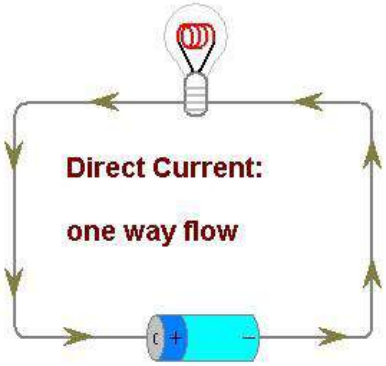
Current is defined to be the amount of charge that passes a given point in a given amount of time

$$I = \frac{dQ}{dt}$$



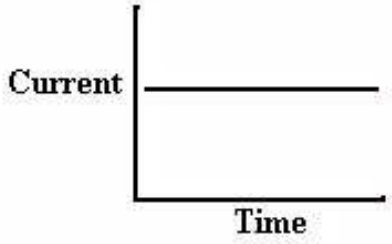
$$I = \frac{q}{t} = \frac{Coloumbs}{Second} = \frac{C}{s} = Amperes = A$$

There are 2 types of Current



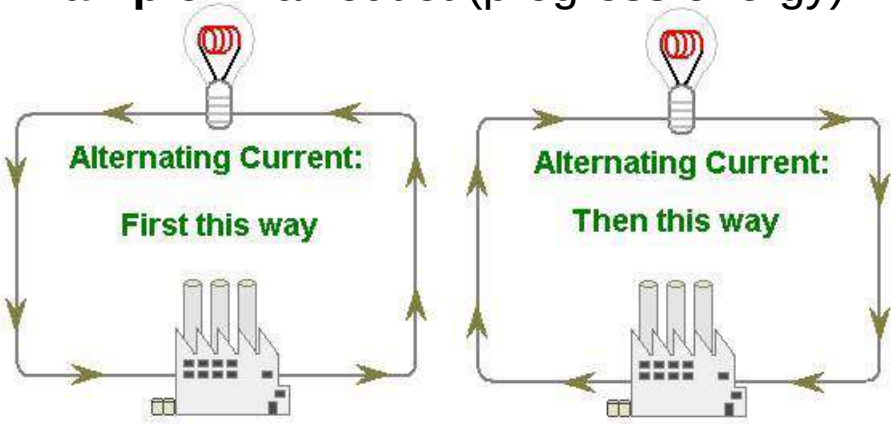
DC = Direct Current - current flows in one direction

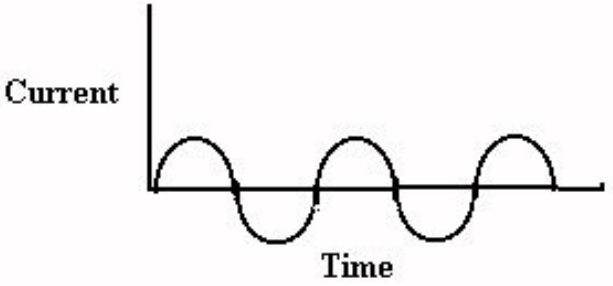
Example: Battery



AC = Alternating Current- current reverses direction many times per second. This suggests that AC devices turn OFF and ON.

Example: Wall outlet (progress energy)





Resistivity

The current density in a wire is not only dependent upon the external electric field that is imposed but

It is also dependent upon the material that is being used

Ohm found that J is proportional to E and in an idealized situation it is directly proportional to E

The resistivity is this proportionality constant and is given by

$$\rho = \frac{E}{J}$$

The greater the resistivity for a given electric field, the smaller the current density

Resistivity

The inverse of resistivity is defined to be the *conductivity*

The resistivity of a material is temperature dependent with the resistivity increasing as the temperature increases

This is due to the increased vibrational motion of the atoms the make up the lattice further inhibiting the motion of the charge carriers

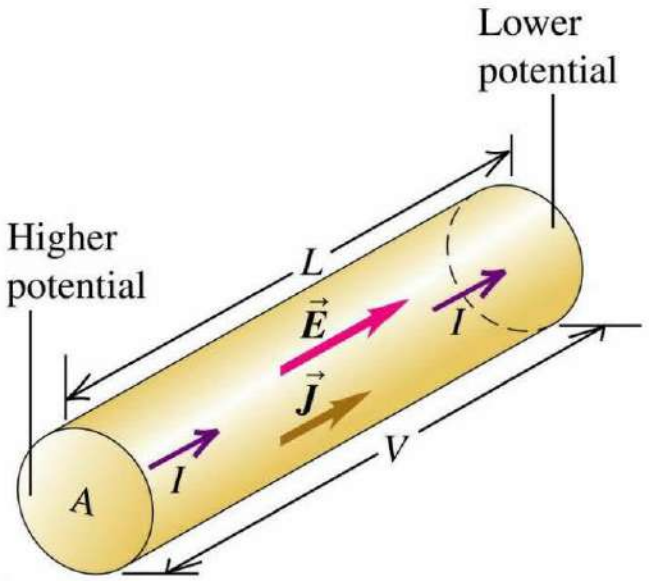
The relationship between the resistivity and temperature is given approximately by

Resistivity at temperature T ..... Temperature coefficient of resistivity

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

Resistivity at reference temperature T_0

Resistivity



Let us take a length of conductor having a certain resistivity

We have that $\vec{E} = \rho \vec{J}$

But *E* and the length of the wire, L, are related to potential difference across the wire by $V = E L$

We also have that $J = \frac{I}{A}$

Putting this all together, we then have

$$\frac{V}{L} = \frac{\rho I}{A} \quad \text{or} \quad V = \frac{\rho L}{A} I$$

Resistance

We take the last equation $V = \frac{\rho L}{A} I$

and rewrite it as $V = I R$

with $R = \frac{\rho L}{A}$ being the resistance

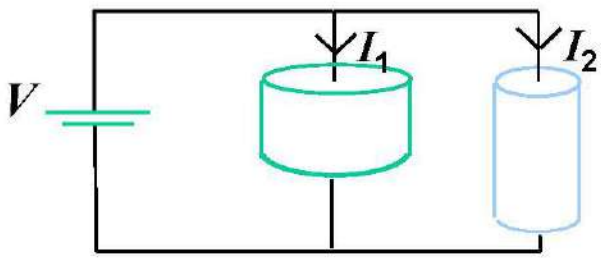
The resistance is proportional to the length of the material and inversely proportional to cross sectional area

$V = I R$ is often referred to as Ohm's Law

The unit for R is the *ohm* or *Volt / Ampere*

Example

Two cylindrical resistors, R_1 and R_2, are made of identical material. R_2 has twice the length of R_1 but half the radius of R_1. These resistors are then connected to a battery V as shown:

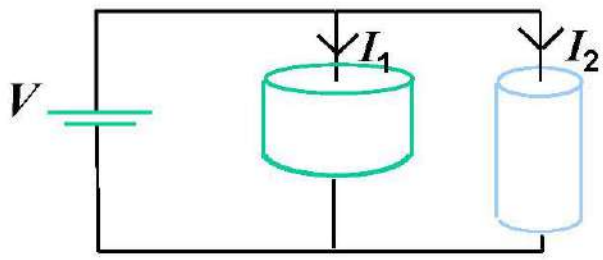


What is the relation between I_1, the current flowing in R_1, and I_2, the current flowing in R_2?

(a) $I_1 < I_2$ (b) $I_1 = I_2$ (c) $I_1 > I_2$

Example

Two cylindrical resistors, R_1 and R_2, are made of identical material. R_2 has twice the length of R_1 but half the radius of R_1. These resistors are then connected to a battery V as shown:



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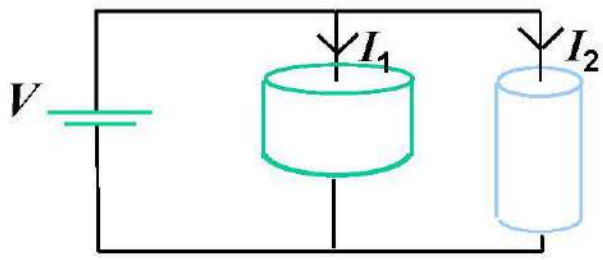
(a) $I_1 < I_2$ (b) $I_1 = I_2$ (c) $I_1 > I_2$

The resistivity of both resistors is the same (r). Therefore the resistances are related as:

$$R_2 = \rho \frac{L_2}{A_2} = \rho \frac{2L_1}{(A_1/4)} = 8\rho \frac{L_1}{A_1} = 8R_1$$

Example

Two cylindrical resistors, R_1 and R_2, are made of identical material. R_2 has twice the length of R_1 but half the radius of R_1. These resistors are then connected to a battery V as shown:



What is the relation between I_1, the current flowing in R_1, and I_2, the current flowing in R_2?

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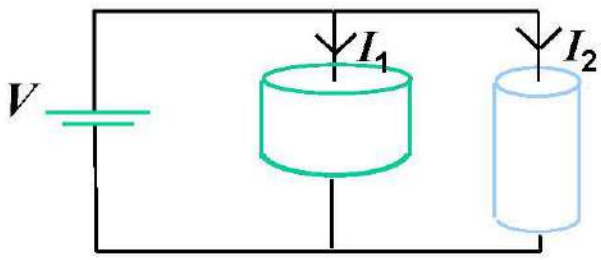
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The resistors have the same voltage across them; therefore

Example

Two cylindrical resistors, R_1 and R_2, are made of identical material. R_2 has twice the length of R_1 but half the radius of R_1. These resistors are then connected to a battery V as shown:



What is the relation between I_1, the current flowing in R_1, and I_2, the current flowing in R_2?

(a) $I_1 < I_2$ (b) $I_1 = I_2$ (c) $I_1 > I_2$

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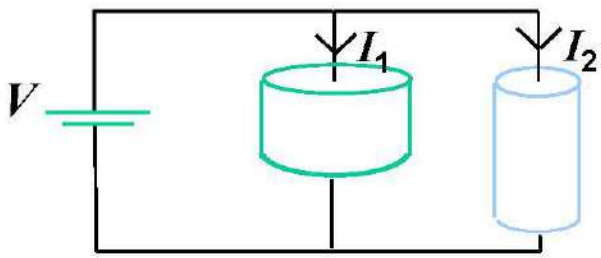
$$R_2 = \rho \frac{L_2}{A_2} = \rho \frac{2L_1}{(A_1/4)} = 8\rho \frac{L_1}{A_1} = 8R_1$$

The resistors have the same voltage across them; therefore

$$I_2 = \frac{V}{R_2} = \frac{V}{8R_1} = \frac{1}{8} I_1$$

Example

Two cylindrical resistors, R_1 and R_2, are made of identical material. R_2 has twice the length of R_1 but half the radius of R_1. These resistors are then connected to a battery V as shown:



What is the relation between I_1, the current flowing in R_1, and I_2, the current flowing in R_2?

(a) $I_1 < I_2$ (b) $I_1 = I_2$ **(c) $I_1 > I_2$**

The resistivity of both resistors is the same (r). Therefore the resistances are related as:

$$R_2 = \rho \frac{L_2}{A_2} = \rho \frac{2L_1}{(A_1/4)} = 8\rho \frac{L_1}{A_1} = 8R_1$$

The resistors have the same voltage across them; therefore

$$I_2 = \frac{V}{R_2} = \frac{V}{8R_1} = \frac{1}{8} I_1$$

Resistance

Because the resistivity is temperature dependent, so is the resistance

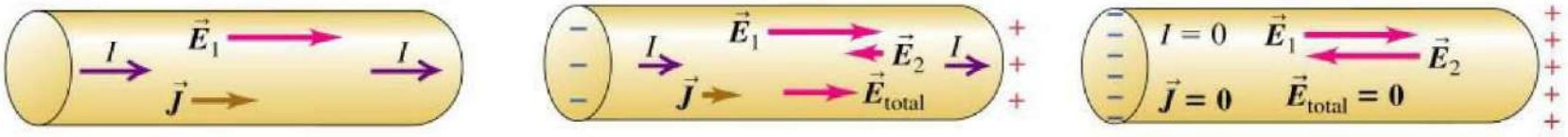
$$R(T) = R_0\left[1 + \alpha(T - T_0)\right]$$

This relationship really only holds if the the length and the cross sectional area of the material being used does not appreciably change with temperature

Electromotive Force

A steady current will exist in a conductor only if it is part of a complete circuit

For an isolated conductor that has an external field impressed on it



Electric field $\vec{E}_1$ produced inside conductor causes current

Current causes charge to build up at ends, producing opposing field $\vec{E}_2$ and reducing current

After a very short time $\vec{E}_2$ has the same magnitude as $\vec{E}_1$: total field $\vec{E}_{\text{total}} = \mathbf{0}$ and current stops completely

Electromotive Force

To maintain a steady current in an external circuit we require the use of a source that supplies electrical energy

Whereas in the external circuit the current flows from higher potential to lower potential, in this source the current must flow from lower potential to higher potential, even though the electrostatic force within the source is in fact trying to do the opposite

In order to do this we must have an *electromotive force, emf,* within such a source

The unit for emf is also Volt

Electromotive Force

Ideally, such a source would have a constant potential difference, $\mathcal{E}$, between its terminals regardless of current

Real sources of emf have an internal resistance which has to be taken into account

The potential difference across the terminals of the source is then given by

$$V_{ab} = \mathcal{E} - I\, r_{\text{internal}}$$

Energy

As a charge "moves" through a circuit, work is done that is equal to

$$qV_{ab}$$

This work does not result in an increase in the kinetic energy of the charge, because of the collisions that occur

Instead, this energy is transferred to the circuit or circuit element within the complete circuit

Power

We usually are not interested in the amount of work done but in the rate at which work is done

This given by $P = V_{ab} I$

If we have a pure resistance, we also have from before that

$$V_{ab} = I R$$

giving us the additional relations

$$P = I^2 R = \frac{V_{ab}^2}{R}$$

Example: A power tool is rated at 9 A when used with a circuit that provides 120-V. What power is used in operating this tool?

$$P = VI = (120\ \text{V})(9\ \text{A})$$

$$P = 1080\ \text{W}$$

Example 6. A 500-W heater draws a current of 10 A. What is the resistance?

$$P = I^2R; \quad R = \frac{P}{I^2} = \frac{500\ \text{W}}{(10\ \text{A})^2}$$

$$R = 5.00\ \Omega$$

Summary of Formulas

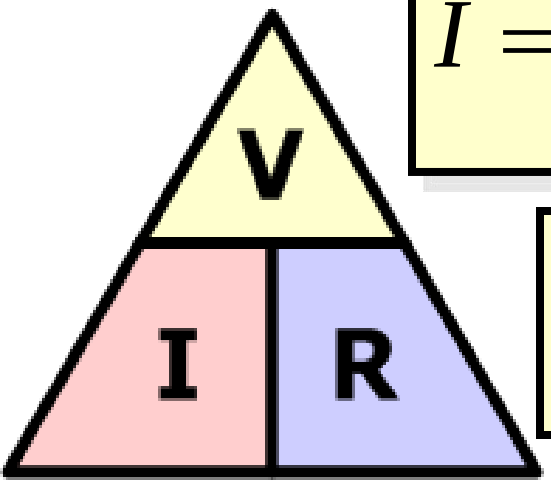
Electric current:

$$I = \frac{Q}{t}$$

$$1\ \text{A} = \frac{1\ \text{C}}{1\ \text{s}}$$

Ohm's Law

$$I = \frac{V}{R}; \quad V = IR; \quad R = \frac{V}{I}$$



Resistance: $1\ \text{ohm} = \dfrac{1\ \text{volt}}{1\ \text{ampere}}$

Summary (Cont.)

Resistivity of materials:

$$R = \rho \frac{L}{A} \quad \text{or} \quad \rho = \frac{RA}{L}$$

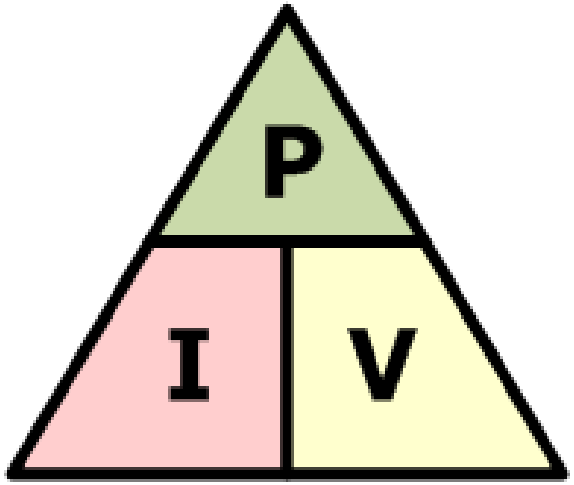
Temperature coefficient of resistance:

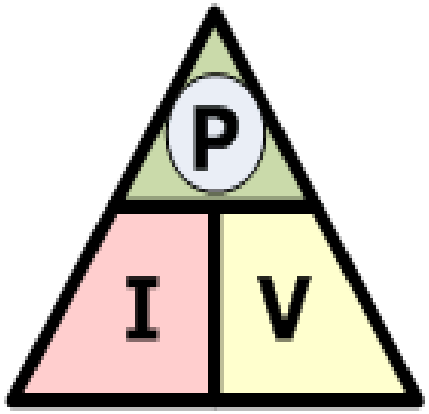
$$\Delta R = \alpha R_0 \Delta t$$

$$\alpha = \frac{\Delta R}{R_0 \Delta t}; \quad \text{Units: } \frac{1}{\text{C}^0}$$

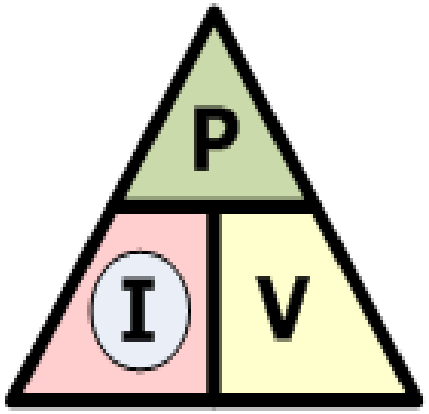
Electric Power *P:*

$$P = VI; \quad P = I^2 R; \quad P = \frac{V^2}{R}$$

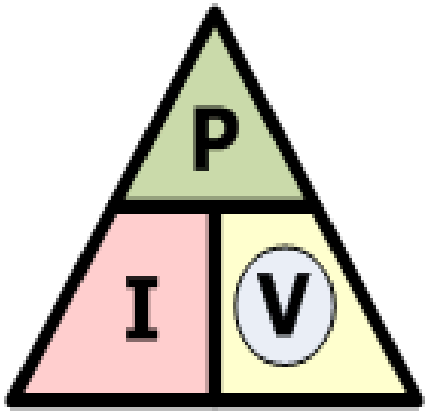




$P = I \times V$

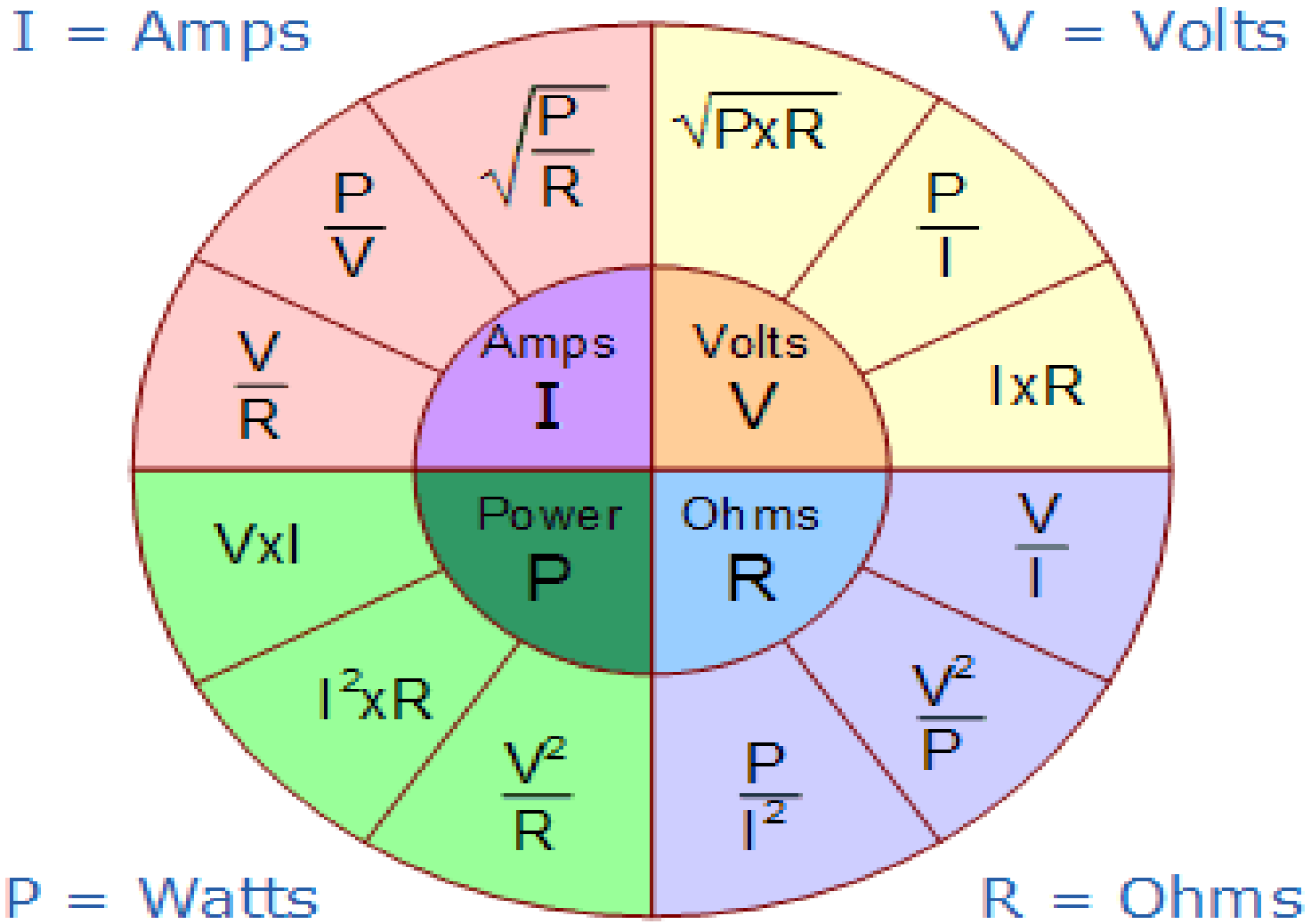


$I = \frac{P}{V}$



$V = \frac{P}{I}$

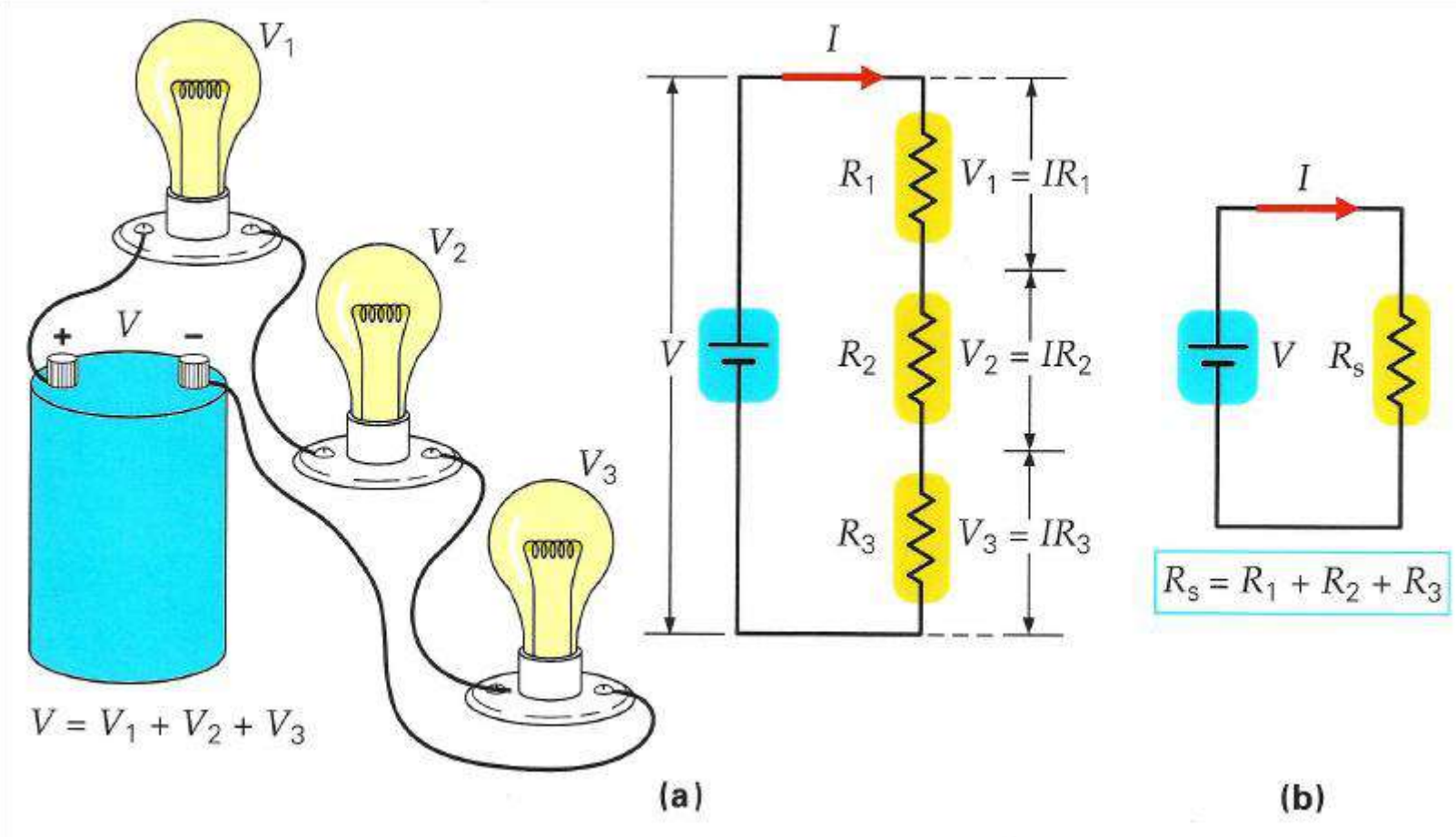
Ohms Law Pie Chart



Ohms Law Matrix Table

Ohms Law Formulas				
Known Values	Resistance (R)	Current (I)	Voltage (V)	Power (P)
Current & Resistance	---	---	$V = I \times R$	$P = I^2 \times R$
Voltage & Current	$R = \frac{V}{I}$	---	---	$P = V \times I$
Power & Current	$R = \frac{P}{I^2}$	---	$V = \frac{P}{I}$	---
Voltage & Resistance	---	$I = \frac{V}{R}$	---	$P = \frac{V^2}{R}$
Power & Resistance	---	$I = \sqrt{\frac{P}{R}}$	$V = \sqrt{P \times R}$	---
Voltage & Power	$R = \frac{V^2}{P}$	$I = \frac{P}{V}$	---	---

Resistors (Light Bulbs) in Series

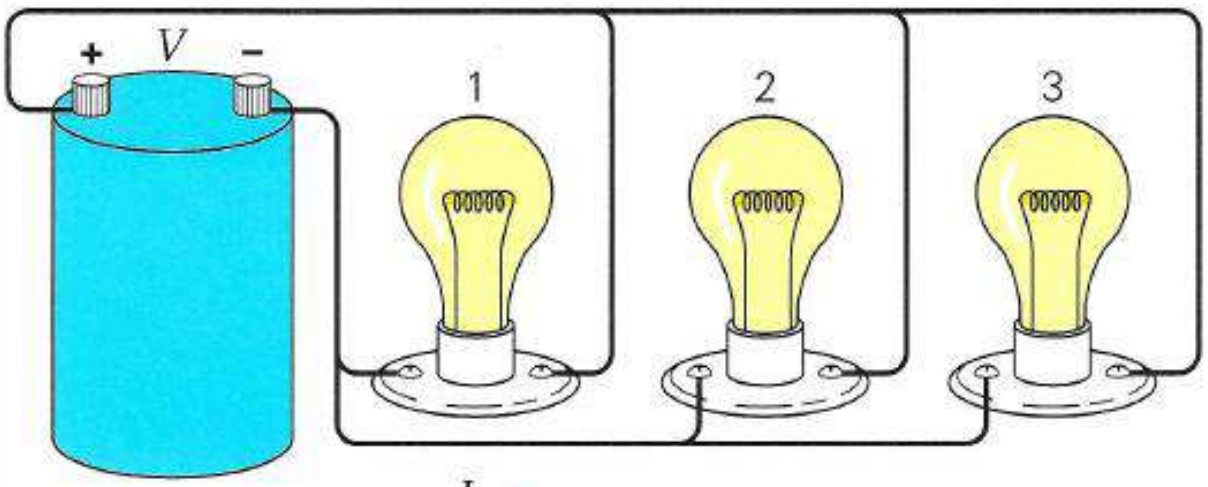


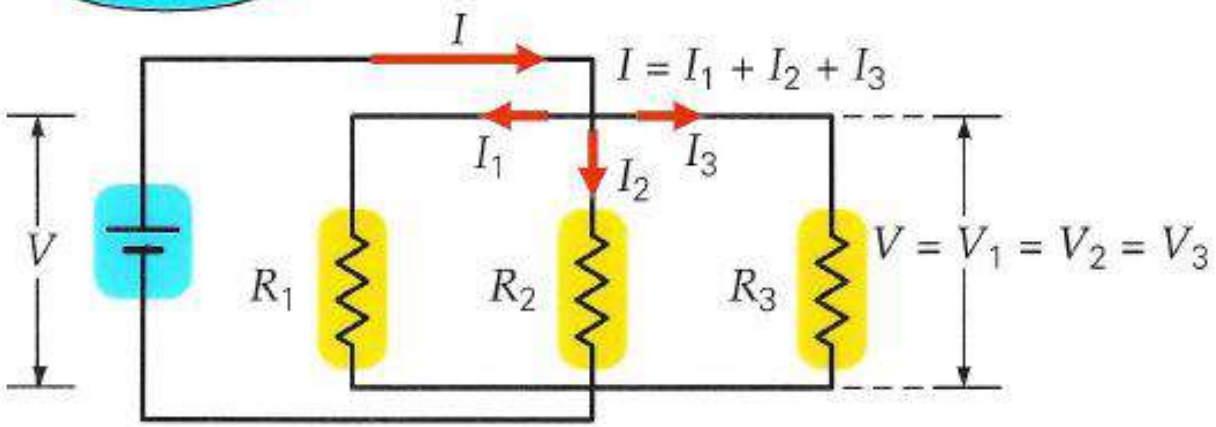
a) When resistors are connected in series, the current through each of them is the same. The sum of the voltage drops across each of them is equal to the voltage of the battery.

b) The equivalent resistance R_s of the resistors in series.

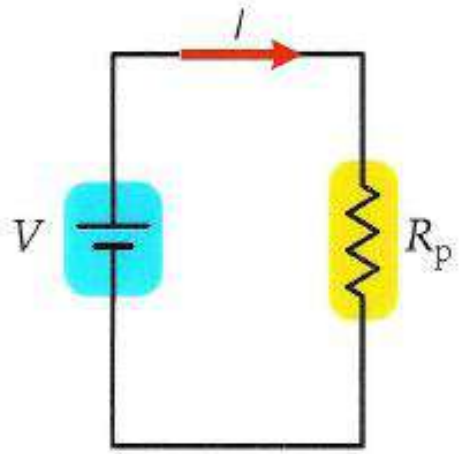
$V = V_1 = V_2 = V_3$

Resistors (Light Bulbs) in Parallel





(a)



$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

(b)

a) When resistors are connected in parallel, the voltage drop across each of the resistors is the same. The current from the battery divides among the resistors.
b) The equivalent resistance R_p of the resistors in parallel.

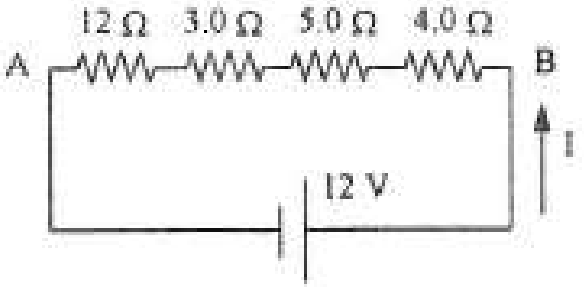
- Series connections provide a way to increase total resistance.
- Parallel connections provide a way to decrease total resistance.

Example 1:

In the circuit shown, find:

(a) the equivalent resistance between points A and B,

(b) the current through each resistor.



Solution:

Given: $R_1 = 12\ \Omega$, $R_2 = 3.0\ \Omega$, $R_3 = 5.0\ \Omega$, $R_4 = 4.0\ \Omega$, $V = 12$ V.

Find: (a) R_s (b) I.

(a) All four resistors are in series combination, so

$$R_s = R_1 + R_2 + R_3 + R_4 = 12\ \Omega + 3.0\ \Omega + 5.0\ \Omega + 4.0\ \Omega = 24\ \Omega.$$

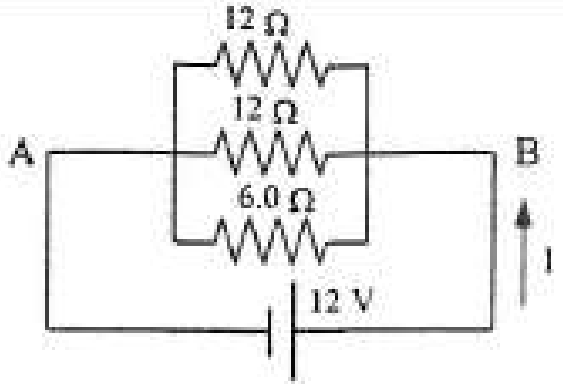
(b) The current through all resistors in series is the same. $I = \dfrac{V}{R} = \dfrac{V}{R_s} = \dfrac{12\text{ V}}{24\ \Omega} = 0.50$ A.

Example 2:

In the circuit shown, find:

(a) the equivalent resistance between points A and B,

(b) the current through the battery.



Solution:

Given: $R_1 = 12\ \Omega$, $R_2 = 12\ \Omega$, $R_3 = 6.0\ \Omega$, $V = 12$ V.

Find: (a) R_p (b) I.

(a) The three resistors are in a parallel combination.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{12\ \Omega} + \frac{1}{12\ \Omega} + \frac{1}{6.0\ \Omega} = \frac{1}{12\ \Omega}\ \frac{1}{12\ \Omega} + \frac{2}{12\ \Omega} = \frac{4}{12\ \Omega} = \frac{1}{3.0\ \Omega}.$$

So $R_p = 3.0\ \Omega$.

(b) From Ohm's law, $I = \frac{V}{R} = \frac{V}{R_p} = \frac{12\ \text{V}}{3.0\ \Omega} = 4.0$ A.

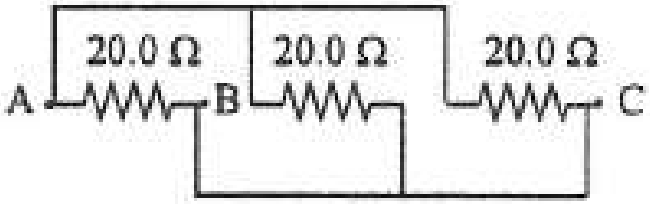
Example 3:

Find the equivalent resistance between points

(a) A and B,

(b) A and C,

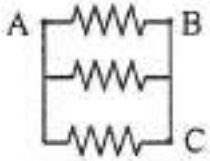
(c) B and C.



Solution: Given: $R_1 = R_2 = R_3 = R = 20.0\ \Omega$.

Find: (a) R_{AB} (b) R_{AC} (c) R_{BC}.

(a) All three resistors are in parallel between points A and B, that is, the ends of all three resistors are across points A and B. We redraw the circuit.



$$\frac{1}{R_{AB}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{3}{R},$$

or $$R_{AB} = \frac{R}{3} = \frac{20.0\ \Omega}{3} = 6.67\ \Omega.$$

(b) Since point C and point B are connected (they are the same point), $R_{AB} = R_{AC} = 6.67\ \Omega$.

(c) Again since points B and C are connected, there is no resistance between them (short circuit).

So $R_{BC} = 0$.

Check for Understanding

1. Which of the following is always the same for resistors in series?
   a) voltage
   b) current
   c) power
   d) energy

   Answer: b

2. Which of the following is always the same for resistors in parallel?
   a) voltage
   b) current
   c) power
   d) energy

   Answer: a

Check for Understanding

3. Are the voltages across resistors in series generally the same?

Answer: No

4. Could they be so in certain cases?

Answer: Yes, if all the resistors are equal

5. Are the currents in resistors in parallel generally the same?

Answer: No

6. Could they be so in certain cases?

Answer: Yes, if all the resistors are equal

Check for Understanding

7. Three resistors that have values of 5 Ω, 2 Ω, and 1 Ω are connected in series to a battery. Which gets the most power, and why?

Answer: The 5 Ω resistor gets the most power because:

→ P=IV

→ all the resistors get the same current

→ the voltage drop is the highest for the largest resistor (V=IR)

8. Three resistors that have values of 5 Ω, 2 Ω, and 1 Ω are connected in parallel to a battery. Which gets the most power, and why?

Answer: The 1Ω resistor gets the most power because:

→ P=IV

→ all the resistors have the same voltage

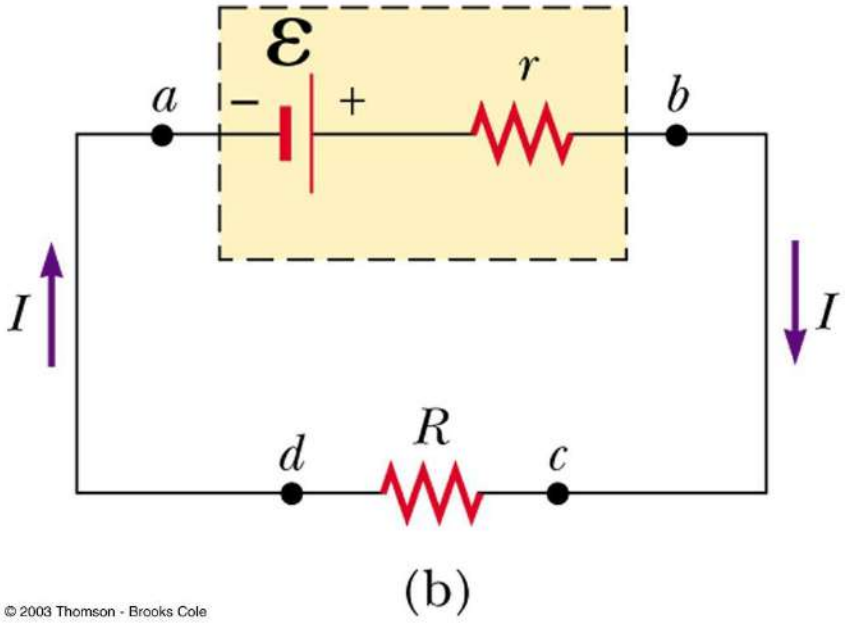
→ it gets the most current

Closed and Open Circuits

- In a **closed circuit** there is a closed-loop path for electrons to follow. A closed circuit must be present for continuous current to exist.
- In an **open circuit** there is not a complete path and therefore no charge flow nor current.

More About Internal Resistance

- The schematic shows the internal resistance, r
- The terminal voltage, $\Delta V = V_b - V_a$
- $\Delta V = \varepsilon - Ir$
- $IR = \varepsilon - Ir$
- For the entire circuit, $\varepsilon = IR + Ir$

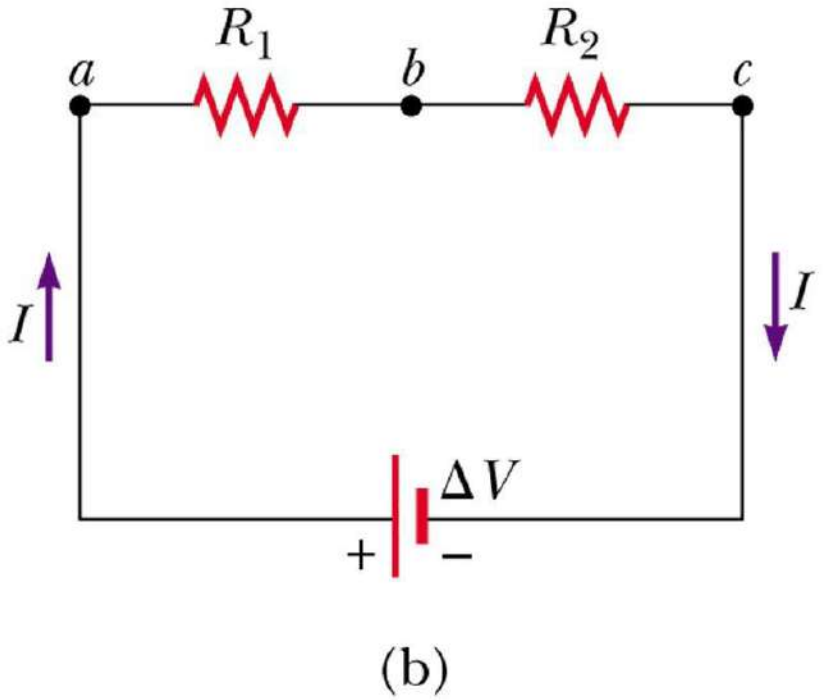


Resistors in Series

- When two or more resistors are connected end-to-end, they are said to be in *series*
- The current is the same in resistors because any charge that flows through one resistor flows through the other
- The sum of the potential differences across the resistors is equal to the total potential difference across the combination

Resistors in Series, cont

- Potentials add
  - $\Delta V = IR_1 + IR_2 = I (R_1+R_2)$
  - Consequence of Conservation of Energy



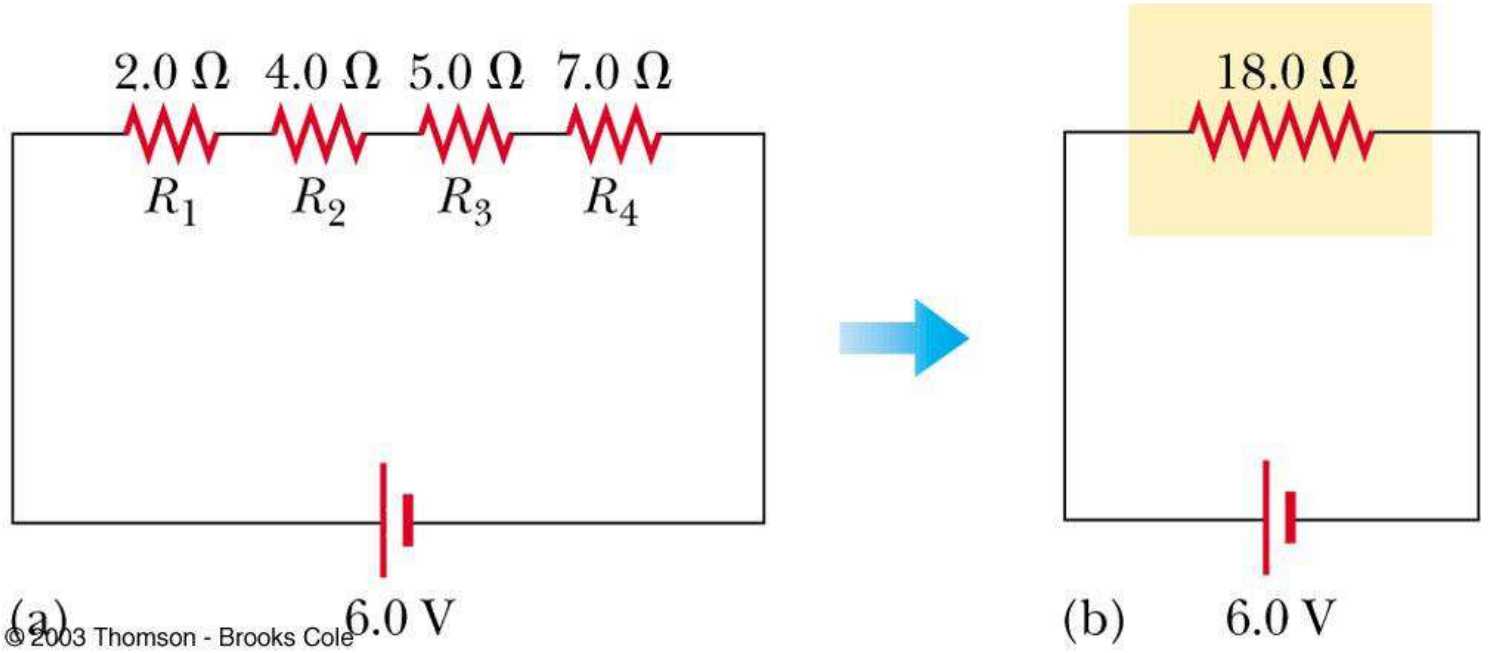
(b)

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Equivalent Resistance – Series

- $R_{eq} = R_1 + R_2 + R_3 + \ldots$
- The equivalent resistance of a series combination of resistors is the algebraic sum of the individual resistances and is always greater than any of the individual resistance

Equivalent Resistance – Series
An Example



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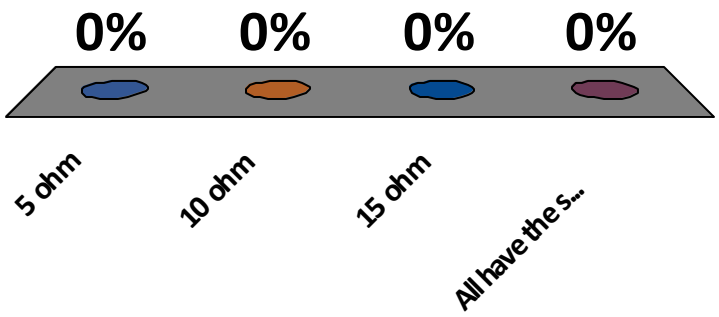
- Four resistors are replaced with their equivalent resistance

Ex. 1

- A 5 ohm, 10 ohm, and 15 ohm resistor are connected in series.
  A) Which resistor has the most current in it?
  B) Which resistor has the largest potential difference?
  C) What is the equivalent (or total) resistance of the circuit?

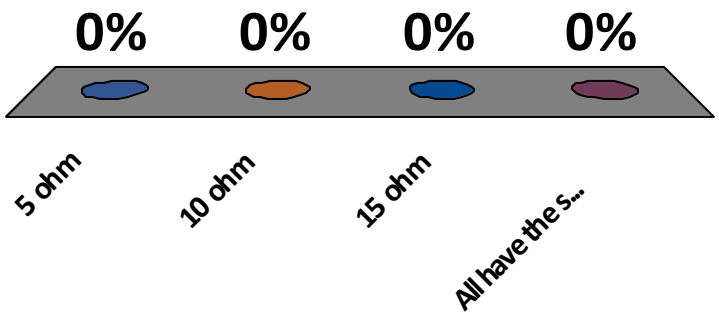
A 5 ohm, 10 ohm, and 15 ohm resistor are connected in series to a battery. Which resistor has the most current passing through it?

1. 5 ohm
2. 10 ohm
3. 15 ohm
4. All have the same current



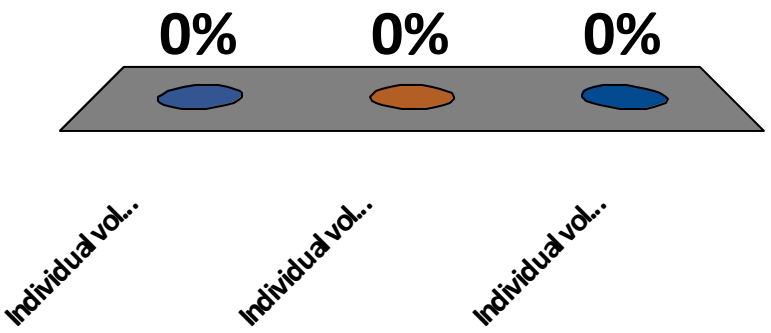
A 5 ohm, 10 ohm, and 15 ohm resistor are connected in series to a battery. Which resistor has the largest potential difference across it?

1. 5 ohm
2. 10 ohm
3. 15 ohm
4. All have the same potential difference



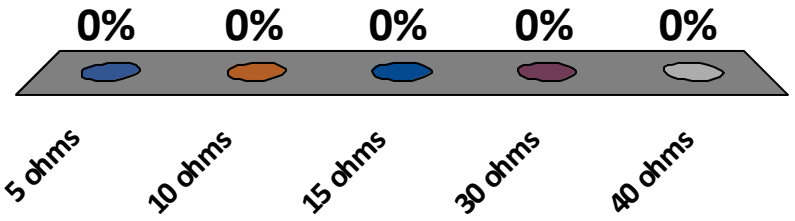
When you add the potential differences across each resistor, what is the relationship to the total voltage?

1. Individual voltages are greater
2. Individual voltages are all equivalent
3. Individual voltages add to equal the total



A 5 ohm, 10 ohm, and 15 ohm resistor are connected in series to a battery. What is the equivalent resistance of the circuit?

1. 5 ohms
2. 10 ohms
3. 15 ohms
4. 30 ohms
5. 40 ohms



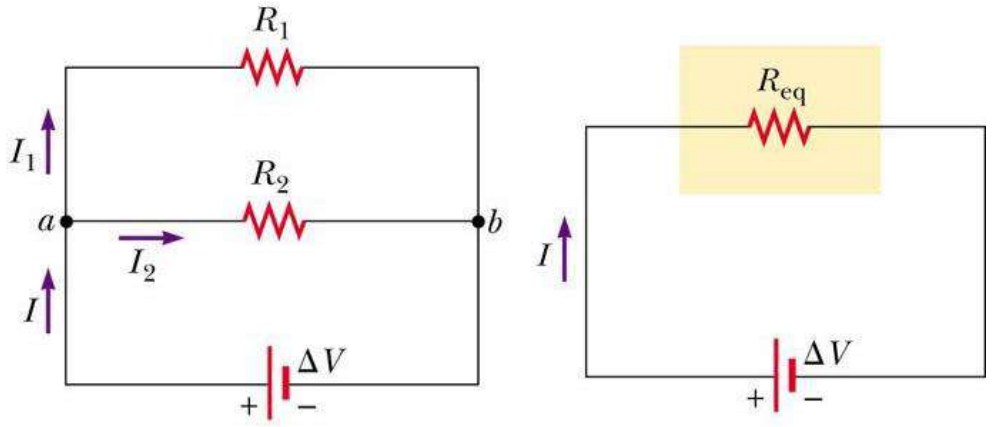
Ex. 2

- Find the current and potential difference across each of the resistors in the following circuits:
  - A) A 2 ohm and a 4 ohm resistor wired in series with a 12V source
  - B) A 4 ohm and a 12 ohm resistor wired in series with a 12V source
  - C) A 150 ohm and a 180 ohm resistor wired in series with a 12V source

Resistors in Parallel

- Analogy: Students leaving an assembly –paths equal doors open, charges equal students
- The more doors that are open, the less the total resistance is
- The more doors that are present mean that less students pass through each door.
- The potential difference across each resistor is the same because each is connected directly across the battery terminals

Equivalent Resistance – Parallel, Examples

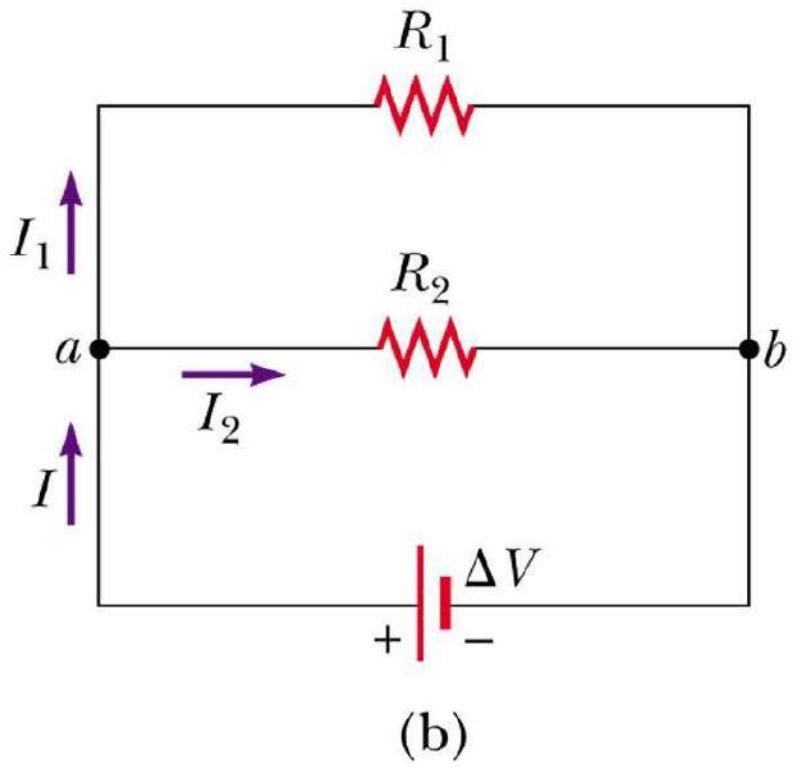


- Equivalent resistance replaces the two original resistances
- *Household circuits* are wired so the electrical devices are connected in parallel
  - Circuit breakers may be used in series with other circuit elements for safety purposes

Equivalent Resistance – Parallel

- Equivalent Resistance

****The equivalent resistance is always less than the smallest resistor in the group



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Ex. 3

- Find the current and potential difference across each of the resistors in the following circuits:
  - A) A 2 ohm and a 4 ohm resistor wired in parallel with a 12V source
  - B) A 4 ohm and a 12 ohm resistor wired in parallel with a 12V source

Ex. 4

- Find the equivalent resistance, the current in each resistor, and the current drawn by the circuit load for a 9.0V battery connected in parallel to the following resistors.
- A) two 30 ohm resistors
- B) three 30 ohm resistors
- C) five 30 ohm resistors

Ex. 5

- Sketch as many different circuits as possible involving a battery and three bulbs of equal resistance.

- How many different circuits would exist with four bulbs?

Problem-Solving Strategy, 3

- A complicated circuit consisting of several resistors and batteries can often be reduced to a simple circuit with only one resistor
  - Replace any resistors in series or in parallel using steps 1 or 2.
  - Sketch the new circuit after these changes have been made
  - Continue to replace any series or parallel combinations
  - Continue until one equivalent resistance is found

Problem-Solving Strategy, 4

- If the current in or the potential difference across a resistor in the complicated circuit is to be identified, start with the final circuit found in step 3 and gradually work back through the circuits
  - Use $\Delta V = I R$ and the procedures in steps 1 and 2

Electrical Safety

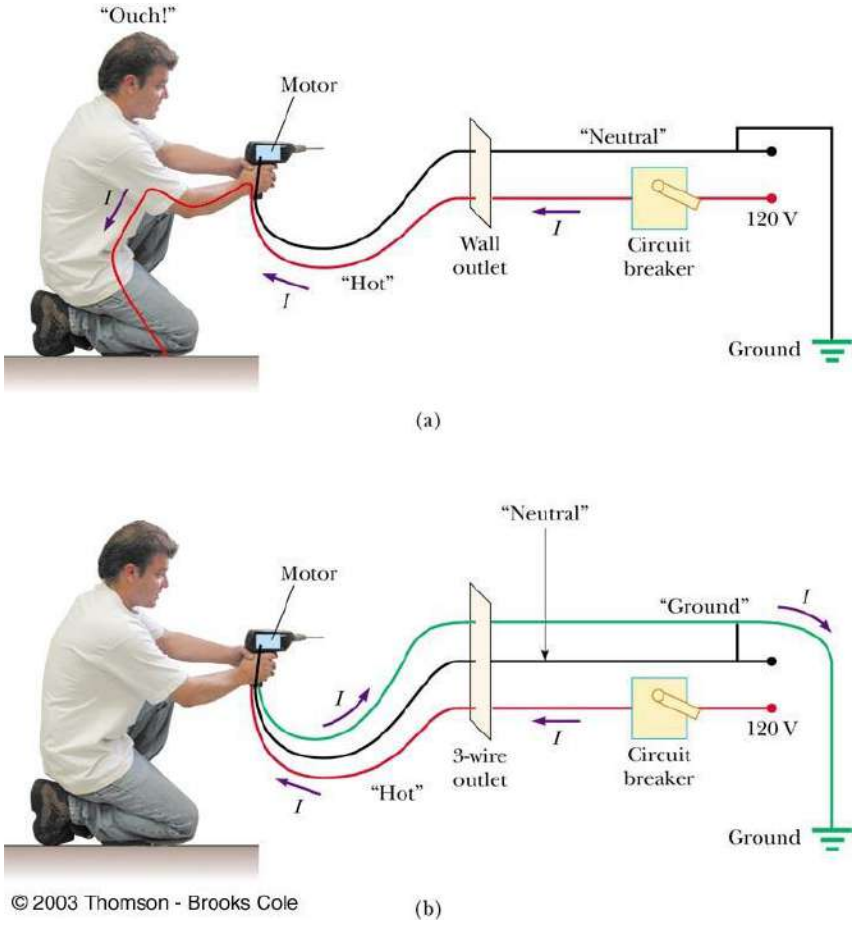
- Electric shock can result in fatal burns
- Electric shock can cause the muscles of vital organs (such as the heart) to malfunction
- The degree of damage depends on
  - the magnitude of the current
  - the length of time it acts
  - the part of the body through which it passes

Effects of Various Currents

- 5 mA or less
  - can cause a sensation of shock
  - generally little or no damage
- 10 mA
  - hand muscles contract
  - may be unable to let go a of live wire
- 100 mA
  - if passes through the body for 1 second or less, can be fatal

Ground Wire

- Electrical equipment manufacturers use electrical cords that have a third wire, called a ground
- Prevents shocks



Ground Fault Circuit Interrupters (GFCI)

- Special power outlets
- Used in hazardous areas
- Designed to protect people from electrical shock
- Senses currents (of about 5 mA or greater) leaking to ground
- Shuts off the current when above this level

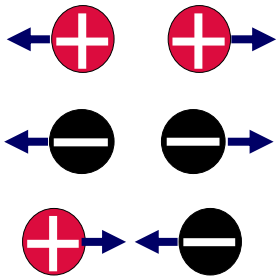
Electricity & Magnetism

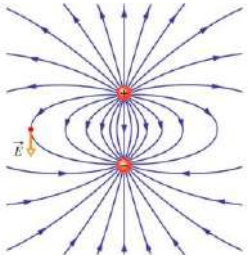
Magnetic Fields and Magnetic Forces

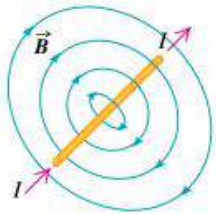
Electric Field & Magnetic Field

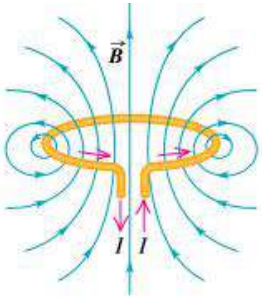
- Electric forces acting at a distance through electric field.
- Vector field, E.
- Source: electric charge.
- Positive charge (+) and negative charge (-).
- Opposite charges attract, like charges repel.
- Electric field lines visualizing the direction and magnitude of E.

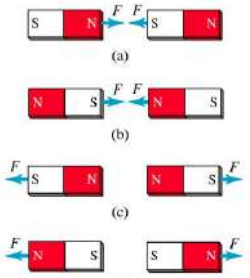
- Magnetic forces acting at a distance through Magnetic field.
- Vector field, B
- Source: moving electric charge (current or magnetic substance, such as permanent magnet).
- North pole (N) and south pole (S)
- Opposite poles attract, like poles repel.
- Magnetic field lines visualizing the direction and magnitude of B.

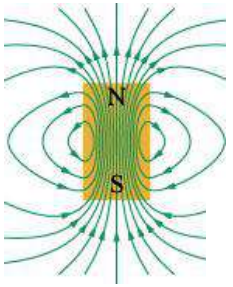












Definition of $\vec{B}$

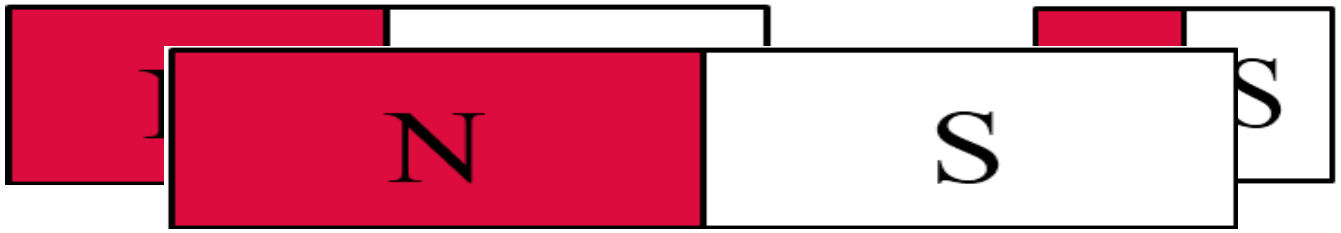
Test charge and electric field

$$\vec{E} = \frac{\vec{F}_E}{q}$$

- Test monopole and magnetic field ?

$$\vec{E} = \frac{\vec{F}_B}{P}$$

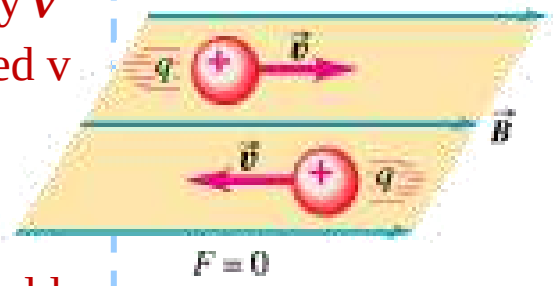
- Magnetic poles are always found in pairs. A single magnetic pole has never been isolated.

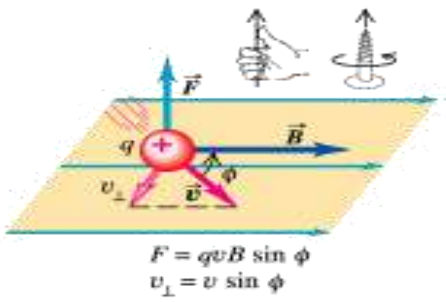


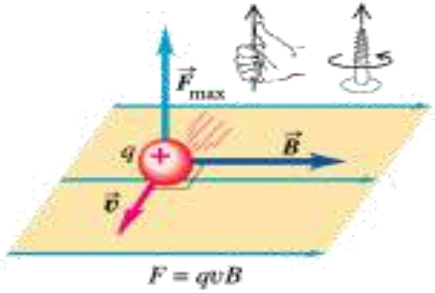


Definition of $\vec{B}$

- Define ***B*** at some point in space in terms of the magnetic force F_B that the field exerts on a ***charged*** particle ***moving*** with a velocity ***v***
- The magnitude F_B is proportional to the charge q and to the speed v of the particle.
- when the charged particle moves parallel to the magnetic field vector $F_B = 0$.
- When velocity vector makes any angle $\theta \neq 0$ with the magnetic field, $\boldsymbol{F_B}$ is perpendicular to both ***B*** and ***v***.
- The magnitude F_B is proportional to $\sin\theta$.







Magnetic Forces

Charged particles experience an electric force when in an electric field regardless of whether they are moving or not moving

There is another force that charged particles can experience even in the absence of an electric field but only when they are motion

A Magnetic Force

Quick Note on Magnetic Fields

Like the electric field, the magnetic field is a *Vector*, having both direction and magnitude

We denote the magnetic field with the symbol $\vec{B}$

The unit for the magnetic field is the *tesla*

$$1\,\textbf{tesla} = 1\mathbf{T} = 1\,N/A\cdot m$$

There is another unit that is also used and that is the *gauss*

$$1\,\textbf{gauss} = 10^{-4}\mathbf{T}$$

Unlike Electric Fields which begin and end on charges, Magnetic Fields have neither a beginning nor an end

Magnetic Forces

Given a charge *q* moving with a velocity *v* in a magnetic field, it is found that there is a force on the charge

This force is

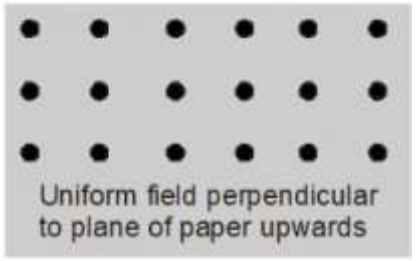
- **proportional to the charge *q***
- **proportional to the speed *v***
- **perpendicular to both *v* and B**
- **proportional to sinϕ where ϕ is the angle between *v* and B**

This can be summarized as $\vec{F} = q\vec{v} \times \vec{B}$

This is the *cross product* of the velocity vector of the charged particle and the magnetic field vector

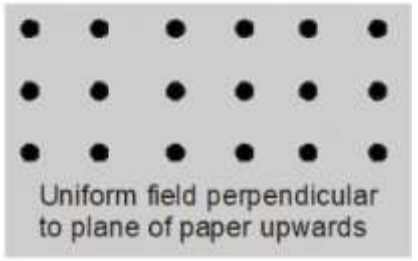
Plotting Field Lines

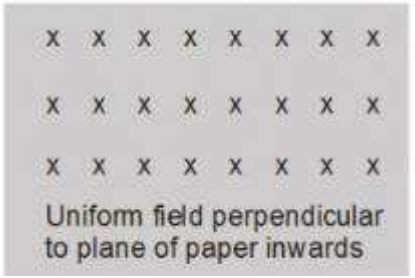
A large dot (•) indicates the tip of the vector when it points out of the plane



Plotting Field Lines

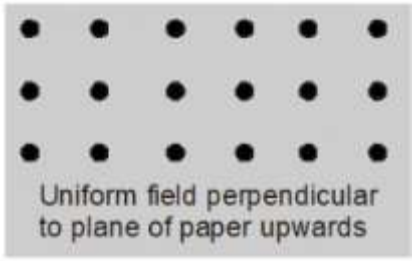
- A large dot (•) indicates the tip of the vector when it points out of the plane
- A cross (×) denotes the tails of the vector when it points into the plane

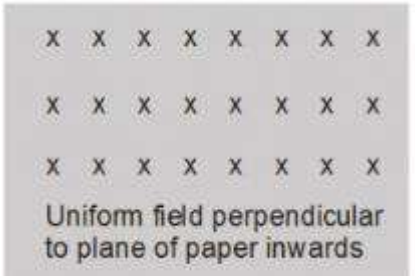


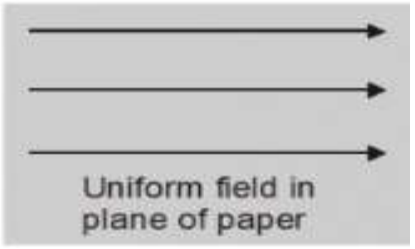


Plotting Field Lines

- A large dot (•) indicates the tip of the vector when it points out of the plane
- A cross (×) denotes the tails of the vector when it points into the plane
- An arrow(→) indicates the tip of the vector when it throw the plane



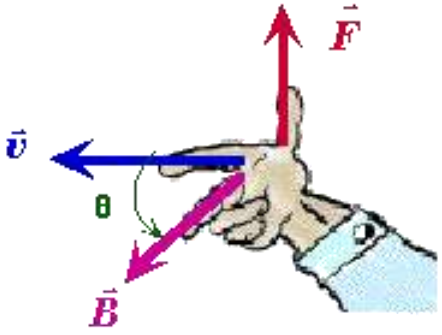


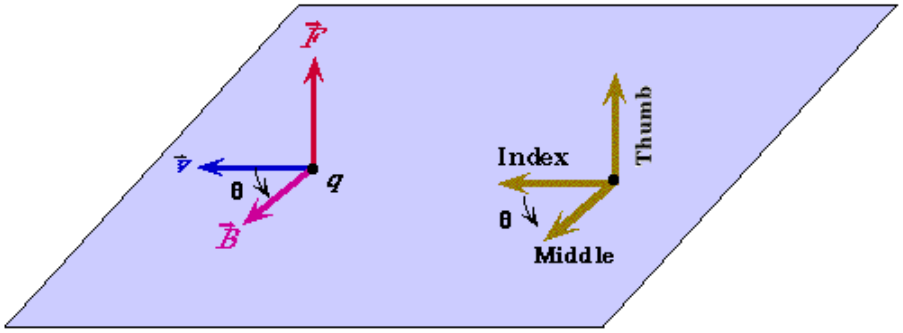


Right Hand Rule

To get the resultant direction for the force do the following:

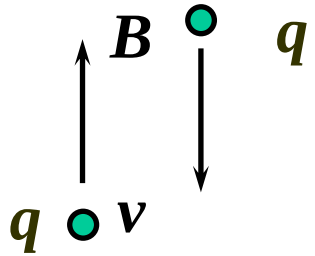
1. **Point your index finger (and your middle finger) along the direction of motion of the charge v**
2. **Rotate your middle finger away from your index finger by the angle θ between v and B**
3. **Hold your thumb perpendicular to the plane formed by both your index finger and middle finger**
4. **Your thumb will then point in the direction of the force F if the charge q is positive**
5. **For q < 0, the direction of the force is opposite your thumb**





Magnetic Forces

There is no force if *v* and B are either parallel or antiparallel

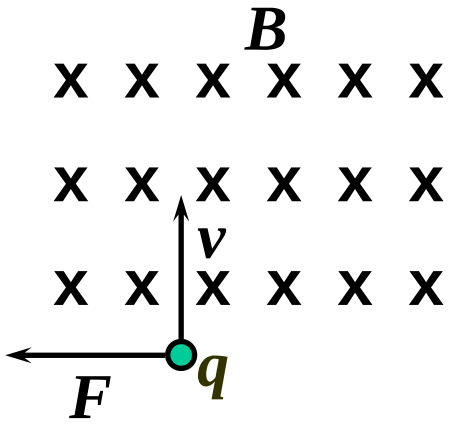


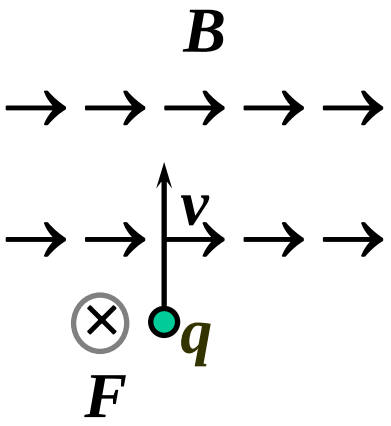
Sin(0) = Sin(180) = 0

$F = 0$

The force is maximum when *v* and B are perpendicular to each other

Sin(90) = 1

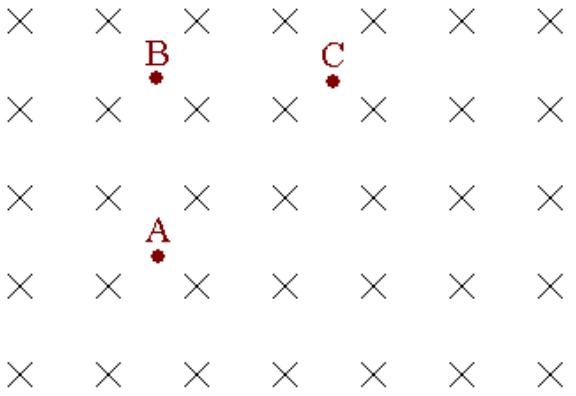




The force on a *negative charge* is in the *opposite direction*

Example

Three points are arranged in a uniform magnetic field. The magnetic field points into the screen.



1) A positively charged particle is located at point A and is stationary. The direction of the magnetic force on the particle is:

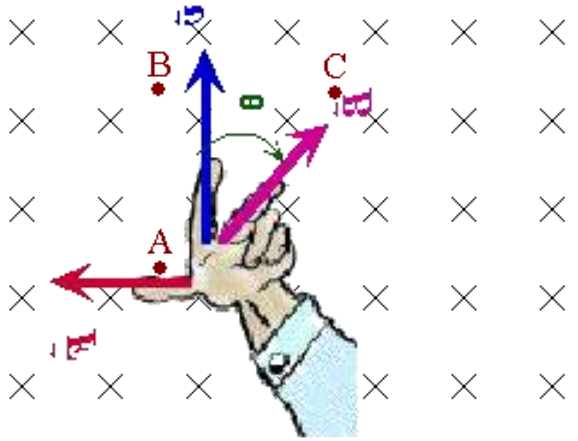
a) Right b) Left c) Into the screen

d) Out of the screen e) Zero

The magnetic force is given by $\vec{F} = q\vec{v} \times \vec{B}$

But *v* is zero. Therefore the force is also zero.

Example

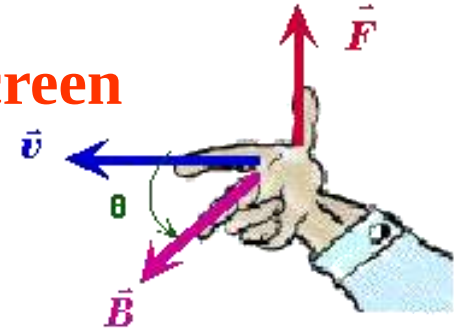


Three points are arranged in a uniform magnetic field. The magnetic field points into the screen.

2) The positive charge moves from point A toward B. The direction of the magnetic force on the particle is:

a) Right b) Left c) Into the screen

d) Out of the screen e) Zero

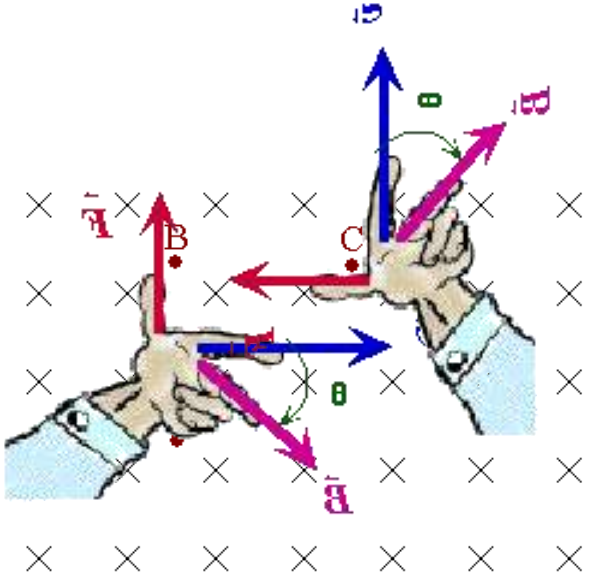


The magnetic force is given by $\vec{F} = q\vec{v} \times \vec{B}$

The cross product of the velocity with the magnetic field is to the left and since the charge is positive the force is then to the left

Example

Three points are arranged in a uniform magnetic field. The magnetic field points into the screen.



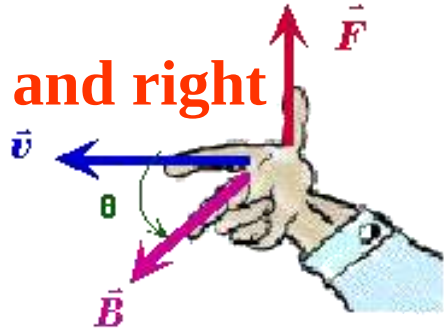
3) The positive charge moves from point A toward C. The direction of the magnetic force on the particle is:

a) up and right

b) up and left

c) down and right

d) down and left



The magnetic force is given by $\vec{F} = q\vec{v} \times \vec{B}$

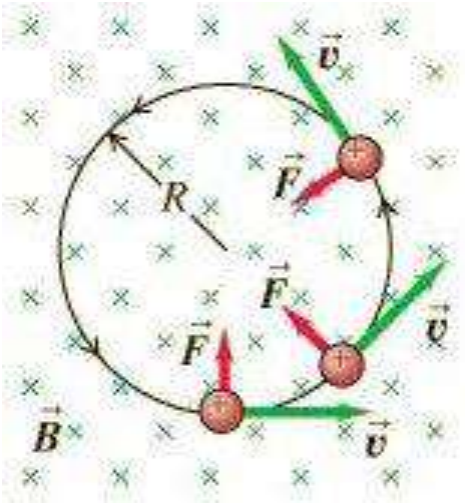
The cross product of the velocity with the magnetic field is to the upper left and since the charge is positive the force is then to the upper left

Motion due to a Magnetic Force

When a charged particle moves in a magnetic field it experiences a force that is perpendicular to the velocity

Since the force is perpendicular to the velocity, the charged particle experiences an acceleration that is perpendicular to the velocity

The magnitude of the velocity does not change, but the direction of the velocity does produce circular motion



The magnetic force does no work on the particle

Motion due to a Magnetic Force

The magnetic force produces circular motion with the centripetal acceleration being given by

$$v^2 / R$$

where R is the radius of the orbit

Using Newton's second law we have $F = qvB = m\dfrac{v^2}{R}$

The radius of the orbit is then given by $R = \dfrac{mv}{qB}$

The angular speed ω is given by $\omega = \dfrac{v}{R} = \dfrac{qB}{m}$

Motion due to a Magnetic Force

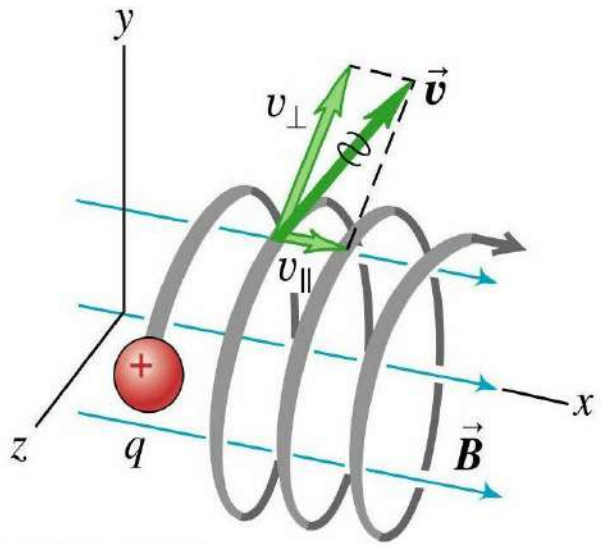
What is the motion like if the velocity is not perpendicular to B?

We break the velocity into components along the magnetic field and perpendicular to the magnetic field

The component of the velocity perpendicular to the magnetic field will still produce circular motion

The component of the velocity parallel to the field produces no force and this motion is unaffected

The combination of these two motions results in a helical type motion



- A particle with charge **2×10^{-6} C** is moving with a velocity of **4×10^{5} m/s** in a uniform **0.5 T** magnetic field, perpendicular to the field. Calculate the magnetic force acting on the particle.

  Solution:

  q= 2×10^{-6} C, v =4×10^{5} m/s, B= 0.5 T

  F = q v B

  Substituting values:

  F = (2×10^{-6} C) × (4×10^{5} m/s) × (0.5 T) = 0.4 N

- If the velocity of a charged particle in a magnetic field double while the field strength remains the same, how does the radius of its circular motion change?
- Answer:

  ∵ the velocity is double $v = 2v1$

  The radius of the orbit is then given by $R = \frac{mv}{qB}$

$$R = \frac{m2v1}{qB} = \frac{2mv1}{qB} = 2R1$$

  So the new radius is **twice the original radius**.

- A proton is moving in a uniform magnetic field of strength **0.5 T** with a velocity of **3×10^{6} m/s** perpendicular to the field. Calculate the radius of its circular motion. (Use (q = 1.6×10^{-19} C), and (m = 1.67×10^{-27} kg).)

  Solution:

  B= 0.5 T, v =**3×10^{6} m/s,** R=?, q = 1.6 × 10-19 C, m = 1.67×10^{-27} kg,

  $\boldsymbol{R = \frac{mv}{qB}}$

  Substituting values:

$$\boldsymbol{R = \frac{1.67 \times 10^{-27} \times 3 \times 10^{6}}{1.6 \times 10^{-19} \times 0.5} = 0.0626\, m}$$

- A charged particle moves in a uniform magnetic field. Explain why the magnetic force does no work on the particle.

  Since the force is perpendicular to motion:

  W = F x cosθ

  ∵ θ=90°

  W = F x cos90=0

  Thus, the force does no work.

- A charged particle with a charge of **5 × 10⁻⁶ C** moves with a velocity of **100 m/s** in a **0.4 T** magnetic field. If the velocity is perpendicular to the field, what is the force acting on the particle?

  Solution:

  $q = 5 \times 10^{-6}$ C, v =100 m/s, B= 0.2 T, R=?,

  $F = q\ v\ B = 5 \times 10^{-6} \times 100 \times 0.4 = 2 \times 10^{-4}$ N

- A proton enters a **0.5 T** magnetic field at a right angle to the field lines. What happens to the direction of its motion?
- Answer:

  Since the proton enters perpendicular to the field, it follows **circular motion** governed by:

  F = q v B

  Its velocity changes **direction** but not magnitude.
- Does a magnetic force change the speed of a particle moving perpendicular to the field? Why or why not
- Answer:

  The force acts **perpendicular** to the velocity, causing direction change but not affecting speed:

  [F = q v B]

  Since work (W = 0), speed remains **constant**.

- Two particles with the same charge but different masses enter the same magnetic field with the same velocity. Which one has the larger radius of motion?

  Answer:

  ∵ they have different masses that's mean m1>m2 as we assume

  The radius of the orbit is then given by $R = \frac{mv}{qB}$

  The particle with the **greater mass** will have the **larger radius** of motion, because a higher mass increase (R)